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Minimal E_6 Supersymmetric Standard Model

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Abstract

We propose a Minimal E_6 Supersymmetric Standard Model (ME$_6$SSM) which allows Planck scale unification, provides a solution to the μ problem and predicts a new Z'. Above the conventional GUT scale $M_{GUT} \sim 10^{16}$ GeV the gauge group corresponds to a left-right symmetric Supersymmetric Pati-Salam model, together with an additional $U(1)_\psi$ gauge group arising from an E_6 gauge group broken near the Planck scale. Below M_{GUT} the ME$_6$SSM contains three reducible **27** representations of the Standard Model gauge group together with an additional $U(1)_X$ gauge group, consisting of a novel and non-trivial linear combination of $U(1)_\psi$ and two Pati-Salam generators, which is broken at the TeV scale by the same singlet which also generates the effective μ term, resulting in a new low energy Z' gauge boson. We discuss the phenomenology of the new Z' gauge boson in some detail.

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1 Introduction

The question of unification of all the forces of Nature is one of the most bold and intriguing in all of physics. It would appear to be somewhat esoteric or premature but for the possibility that that discoveries at the CERN LHC may provide an unprecedented opportunity to shed light on this question. For example, it is well known that, without new physics, the electroweak and strong gauge couplings extracted from LEP data and extrapolated to high energies using the renormalisation group (RG) evolution do not meet within the Standard Model (SM), so unification appears to require some new physics. One example of new physics that can lead to unification is TeV scale supersymmetry (SUSY). For example, in the minimal supersymmetric standard model (MSSM) [1], the gauge couplings converge to a common value at a high energy scale $M_{GUT} \sim 10^{16}$ GeV, allowing some supersymmetric Grand Unified Theory (GUT) to emerge above this scale. Thus evidence for the MSSM at the LHC would provide support for unification at M_{GUT}.

However, despite its obvious attractions, the standard paradigm of SUSY GUTs based on the MSSM faces some serious shortcomings. On the one hand, the failure to discover superpartners or the Higgs boson by the LEP and the Tevatron indicates that the scale of SUSY breaking must be higher than previously thought, leading to fine-tuning at the per cent level. On the other hand experimental limits on proton decay and the requirement of Higgs doublet-triplet splitting provides some theoretical challenges at the high scale [2–4]. Related to the doublet-triplet splitting problem is the origin of μ, the SUSY Higgs and Higgsino mass parameter, which from phenomenology must be of order the SUSY breaking scale, but which *a priori* is independent of the SUSY breaking scale. Apart from these problems, unification of gauge couplings near $M_{GUT} \sim 10^{16}$ GeV leaves open the question of a full unification of all the forces with gravity, although this may be achieved in the framework of string unification, including high energy threshold effects [5].

The challenges facing SUSY GUTs based on the MSSM motivate alternative approaches that successfully overcome these problems. Recently an E_6 Supersymmetric Standard Model (E$_6$SSM) has been proposed [6, 7], in which the low energy particle content consists of three irreducible **27** representations of the gauge group E_6 plus, in addition, a pair of non-Higgs doublets $H', \overline{H}'$ arising from incomplete $\mathbf{27}', \overline{\mathbf{27}}'$ representations. In the E$_6$SSM gauge coupling unification works very well, even better than in the MSSM [8]. Moreover, the E$_6$SSM also solves the μ problem via a singlet coupling to two Higgs doublets. In the E$_6$SSM a special role is played by a low energy gauged $U(1)_N$ symmetry, which consists of a particular linear combination of Abelian generators contained in the $SU(5)$ breaking chain of E_6. The $U(1)_N$ results from GUT scale Higgs which develop vacuum expectation values (VEVs) in their right-handed neutrino components. As a consequence the right-handed neutrino carries zero charge under $U(1)_N$, thereby allowing a conventional see-saw mechanism via heavy right-handed Majorana masses. The $U(1)_N$ is anomaly-free since the low energy theory contains complete $\mathbf{27}'s$ (minus the heavy chargeless right-handed neutrinos) and $H', \overline{H}'$, which have opposite $U(1)_N$ charges. The $U(1)_N$ also serves to forbid the μ term, but allow the singlet cou-

pling, and is broken by the singlet VEV which generates the effective μ term. Without the gauged $U(1)_N$ symmetry there would be a Peccei-Quinn global symmetry of the theory resulting in an unwanted Goldstone boson. In the next-to-MSSM (NMSSM) this global symmetry is broken explicitly by a cubic singlet term, however this only reduces the symmetry to Z_3 and so the singlet VEV then leads to dangerous cosmological domain walls. In the E$_6$SSM, with a gauged $U(1)_N$ symmetry, the cubic singlet term is forbidden and the resulting would-be Goldstone boson of the theory is eaten by the Higgs mechanism to produce a massive Z' gauge boson.

However, although the E$_6$SSM solves the μ problem, the presence of the non-Higgs doublets $H', \overline{H}'$ cannot be regarded as completely satisfactory since it introduces a new μ' problem and in this case a singlet coupling generating the mass μ' for $H', \overline{H}'$ is not readily achieved [6, 7]. Similarly, although the E$_6$SSM solves the usual doublet-triplet splitting problem, since the usual Higgs doublets are contained along with colour triplets in complete low energy **27** representations, the presence of the low energy $H', \overline{H}'$ introduces a new doublet-triplet splitting problem because their triplet partners are assumed to be very heavy. However, since the only purpose of including the non-Higgs states $H', \overline{H}'$ is to help achieve gauge coupling unification at $M_{GUT} \sim 10^{16}$ GeV, it is possible to consider not introducing $H', \overline{H}$ at all, thereby allowing a solution to the μ problem and the doublet-triplet splitting problem, without re-introducing new ones. An immediate objection to removing the non-Higgs states $H', \overline{H}'$ of the E$_6$SSM is that the gauge couplings will no longer converge at $M_{GUT} \sim 10^{16}$ GeV, so at first sight this possibility looks unpromising. In a recent paper we showed how this objection could be overcome by embedding the theory into a left-right symmetric Pati-Salam theory at $M_{GUT} \sim 10^{16}$ GeV, leading to a unification of all forces with gravity close to the Planck scale [9]. However, the analysis did not include the effects of an additional low energy $U(1)'$ gauge group that would be required for a consistent resolution of the μ problem.

The purpose of the present paper is to propose a Minimal E_6 Supersymmetric Standard Model (ME$_6$SSM) based on three low energy reducible **27** representations of the Standard Model gauge group which allows Planck scale unification and provides a solution to the μ problem and doublet-triplet splitting problem, without re-introducing either of these problems. Above the conventional GUT scale the ME$_6$SSM is embedded into a left-right symmetric Supersymmetric Pati-Salam model with an additional $U(1)_\psi$ gauge group arising from an E_6 gauge group broken near the Planck scale. In our previous analysis [9] we assumed for simplicity that the $U(1)_\psi$ gauge group was broken at the Planck scale. Here we assume that $U(1)_\psi$ remains unbroken down to M_{GUT} and that below M_{GUT} an additional $U(1)_X$ gauge group, consisting of a novel and non-trivial linear combination of $U(1)_\psi$ and two Pati-Salam generators, survives down to low energies. Eventually $U(1)_X$ is broken at the TeV scale by the same singlet that also generates the effective μ term, resulting in a new low energy Z' gauge boson. We discuss the phenomenology of the new Z' gauge boson in some detail. We compare the Z' of the ME$_6$SSM (produced via the Pati-Salam breaking chain of E_6, where E_6 is broken at the Planck scale) to the Z' of the E$_6$SSM (from the $SU(5)$ breaking chain of E_6, where E_6 is broken at the GUT scale) and discuss how they can be distinguished by their different couplings, which enable the two models to be resolved experimentally. In the case of the

ME$_6$SSM the Z' gauge boson properties can be said to provide a window on Planck scale physics. In both cases the right-handed neutrinos carry zero charge under the extra low energy $U(1)'$ gauge groups, allowing a conventional see-saw mechanism.

The layout of the rest of this paper is as follows. In the section 2 we consider the pattern of symmetry breaking assumed in this paper. In section 3 we consider the two loop RG evolution of gauge couplings in this model from low energies, through the Pati-Salam breaking scale at $M_{GUT} \sim 10^{16}$ GeV (assuming various Pati-Salam breaking Higgs sectors) and show that the Pati-Salam gauge couplings converge close to the Planck scale $M_p \sim 10^{19}$ GeV. In section 4 we discuss the phenomenology of the new Z' of the ME$_6$SSM and compare it to that of the E$_6$SSM. In section 5 we shall construct an explicit supersymmetric model of the kind we are considering. Finally we conclude the paper in section 5.

2 Pattern of Symmetry Breaking

The two step pattern of gauge group symmetry breaking that we analyse in this paper is:

$$E_6 \overset{M_p}{\overbrace{\longrightarrow}} G_{4221} \otimes D_{LR} \overset{M_{GUT}}{\overbrace{\longrightarrow}} G_{3211} \tag{1}$$

where the gauge groups are defined by:

$$\begin{aligned} G_{4221} &\equiv SU(4) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_\psi, \\ G_{3211} &\equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X \end{aligned} \tag{2}$$

and we have assumed that the first stage of symmetry breaking happens close to the Planck scale and the second stage happens close to the conventional GUT scale. The first stage of symmetry breaking is based on the maximal E_6 subgroup $SO(10) \otimes U(1)_\psi$ and the maximal $SO(10)$ subgroup $G_{422} \otimes D_{LR}$ corresponding to a Pati-Salam symmetry with D_{LR} being a discrete left-right symmetry.[3] The pattern of symmetry breaking assumed in this paper is different from that commonly assumed in the literature based on the maximal $SO(10)$ subgroup $SU(5) \otimes U(1)_\chi$ [6,7,11]. In particular, the Pati-Salam subgroup does not contain the Abelian gauge group factor $U(1)_\chi$. The only Abelian gauge group factor involved in this pattern of symmetry breaking is $U(1)_\psi$, with a low energy $U(1)_X$ emerging along with $U(1)_Y$ below the Pati-Salam symmetry breaking scale.

The first stage of symmetry breaking close to M_p will not be considered explicitly in this paper. We only remark that the Planck scale theory may or may not be based on a higher dimensional string theory. Whatever the quantum gravity theory is, it will involve some high energy threshold effects, which will depend on the details of the high

[3]Under D_{LR} the matter multiplets transform as $q_L \to q_L^c$, and the gauge groups $SU(2)_L$ and $SU(2)_R$ become interchanged [10].

energy theory and which we do not consider in our analysis. Under $E_6 \to G_{4221}$ the fundamental E_6 representation **27** decomposes as:

$$\mathbf{27} \to (4,2,1)_{\frac{1}{2}} + (\overline{4},1,\overline{2})_{\frac{1}{2}} + (1,2,2)_{-1} + (6,1,1)_{-1} + (1,1,1)_2 \tag{3}$$

where the subscripts are related to the $U(1)_\psi$ symmetry's charge assignments [12, 13]. One family of the left-handed quarks and leptons can come from the $(4,2,1)_{\frac{1}{2}}$; one family of the charge-conjugated quarks and leptons, including a charge-conjugated neutrino ν^c, can come from the $(\overline{4},1,\overline{2})_{\frac{1}{2}}$; the MSSM Higgs bosons can come from the $(1,2,2)_{-1}$; two colour-triplet weak-singlet particles can come from the $(6,1,1)_{-1}$; and the $(1,1,1)_2$ is a MSSM singlet.

The second stage of symmetry breaking close to M_{GUT} is within the realm of conventional quantum field theory and requires some sort of Higgs sector, in addition to the assumed matter content of three **27** representations of the gauge group E_6. In order to break the symmetry G_{4221} to G_{3211} at M_{GUT}, the minimal Higgs sector required are the G_{4221} representations $\overline{H}_R = (\overline{4},1,\overline{2})_{\frac{1}{2}}$ and $H_R = (4,1,2)_{-\frac{1}{2}}$.[4] When these particles obtain VEVs in the right-handed neutrino directions $< \overline{H}_R > = < \nu^c_H >$ and $< H_R > = < \nu^H_R >$ they break the $SU(4) \otimes SU(2)_R \otimes U(1)_\psi$ symmetry to $SU(3)_c \otimes U(1)_Y \otimes U(1)_X$. Six of the off-diagonal $SU(4)$ and two of the off-diagonal $SU(2)_R$ fields receive masses related to the VEV of the Higgs bosons. The gauge bosons associated with the diagonal $SU(4)$ generator T_4^{15}, the diagonal $SU(2)_R$ generator T_R^3 and the $U(1)_\psi$ generator T_ψ, are rotated by the Higgs bosons to create one heavy gauge boson and two massless gauge bosons associated with $U(1)_Y$ and $U(1)_X$. The part of the symmetry breaking G_{4221} to G_{3211} involving the diagonal generators is then:

$$U(1)_{T_4^{15}} \otimes U(1)_{T_R^3} \otimes U(1)_\psi \to U(1)_Y \otimes U(1)_X, \tag{4}$$

where the charges of the "right-handed neutrino" component of the Higgs which gets the VEV are:

$$\nu^H_R = \left(-\frac{1}{2}\sqrt{\frac{3}{2}},\ \frac{1}{2},\ -\frac{1}{2}\sqrt{\frac{1}{6}} \right) \tag{5}$$

under the corresponding correctly E_6 normalized diagonal generators:[5]

$$T_4^{15} = \sqrt{\frac{3}{2}}\ diag(\frac{1}{6},\frac{1}{6},\frac{1}{6},-\frac{1}{2}), \quad T_R^3 = \frac{1}{2}\ diag(1,-1), \quad T_\psi/\sqrt{6}. \tag{6}$$

[4] We show in the Appendix that the symmetry breaking G_{4221} to G_{3211} also requires an MSSM singlet S, from a **27** multiplet of E_6, to get a low-energy VEV. The VEV of this MSSM singlet is also used to solve the μ problem.

[5] Note that we choose to normalize the E_6 generators G^a by $Tr(G^aG^b) = 3\delta^{ab}$. It then follows that the Pati-Salam and standard model operators are conventionally normalized by $Tr(T^aT^b) = \frac{1}{2}\delta^{ab}$. The correctly normalized E_6 generator corresponding to $U(1)_\psi$ is $T_\psi/\sqrt{6}$ where T_ψ corresponds to the charges in Eq.3.

In the Appendix we discuss the symmetry breaking in Eq.4 in detail. To simplify the discussion we observe that $T^{15}=\sqrt{\frac{3}{2}}\frac{(B-L)}{2}$ where B and L are the baryon and lepton number assignments of each Standard Model particle. The Higgs charges can then be written as

$$\nu_R^H=\left(-\frac{1}{2},\ \frac{1}{2},\ -\frac{1}{2}\right) \tag{7}$$

under the corresponding generators $T_{B-L}=\frac{B-L}{2}$, T_R^3 and T_ψ. Then it is clear to see why the hypercharge generator Y is preserved by the Higgs H_R and $\overline{H}_R$ since

$$Y=T_R^3+\frac{(B-L)}{2} \tag{8}$$

takes a zero value for the right-handed neutrino and anti-neutrino Higgs components which develop VEVs. However this is not the only Abelian generator that is preserved by this Higgs sector. The Higgs H_R and $\overline{H}_R$ VEVs also preserve the combinations of generators $T_\psi-T_{B-L}$ and $T_\psi+T_R^3$.

As shown in the Appendix, precisely one additional Abelian generator orthogonal to $U(1)_Y$ is preserved, namely:

$$X=(T_\psi+T_R^3)-c_{12}^2Y \tag{9}$$

where $c_{12}=\cos\theta_{12}$ and the mixing angle is given by

$$\tan\theta_{12}=\frac{g_{2R}}{g_{B-L}},\qquad g_{B-L}=\sqrt{\frac{3}{2}}\,g_4, \tag{10}$$

where the E_6 normalized Pati-Salam coupling constants g_{2R} and g_4 are evaluated at the G_{4221} symmetry breaking scale M_{GUT}. Note that this Abelian generator X depends on the values that the Pati-Salam coupling constants take at a particular energy scale. It is easy to prove that it is a general rule that, if three massless gauge fields are mixed, then at least two of the resulting mass eigenstate fields must have a charge that depends on the value of the original gauge coupling constants. See the Appendix for more discussion on this unusual aspect of X.

The "GUT" (in this case E_6) normalized generator is

$$T_X=\frac{1}{N_X}X \tag{11}$$

where the normalization constant N_X is given by:

$$N_X^2\equiv 7-2c_{12}^2+\frac{5}{3}c_{12}^4 \tag{12}$$

	Q	L	u^c	d^c	e^c	ν^c	h_2	h_1	D	$\overline{D}$	S
T_{B-L}	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	0
T_R^3	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
T_ψ	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	-1	-1	-1	2
Y	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{2}{3}$	$\frac{1}{3}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$	0
$T_\psi + T_R^3$	$\frac{1}{2}$	$\frac{1}{2}$	0	1	1	0	$-\frac{1}{2}$	$-\frac{3}{2}$	-1	-1	2

Table 1: This table lists the $T_{B-L} \equiv \frac{B-L}{2}$, T_R^3, T_ψ, hypercharge $Y \equiv T_{B-L} + T_R^3$, and $T_\psi + T_R^3$ charge assignments for the G_{3211} representations of the **27** multiplet of E_6 [12]. The charge for the $U(1)_X$ group is dependent on $X \equiv (T_\psi + T_R^3) - c_{12}^2 Y$ where c_{12}^2 is the square of the cosine of the mixing angle θ_{12} given by $\tan\theta_{12} \equiv \mathbf{g}_{2R}/\mathbf{g}_{B-L}$.

From Eq.9, the corresponding gauge coupling constant g_X^0 may be expressed in terms of the $SU(4)$, $SU(2)_R$ and $U(1)_\psi$ gauge coupling constants g_4, g_{2R} and g_ψ as:

$$\frac{1}{\alpha_X^0} = \frac{1}{\frac{1}{6}\alpha_\psi} + \frac{1}{\frac{3}{2}\alpha_4 + \alpha_{2R}} \tag{13}$$

where $\alpha_X^0 = \frac{(g_X^0)^2}{4\pi}$, $\alpha_{2R} = \frac{g_{2R}^2}{4\pi}$; $\alpha_4 = \frac{g_4^2}{4\pi}$; and $\alpha_\psi = \frac{g_\psi^2}{4\pi}$. In terms of the E_6 normalized generator $T_X = X/N_X$, the normalized gauge coupling constant $g_X = g_X^0 N_X$ so that $\alpha_X = \alpha_X^0 N_X^2$ giving:

$$\frac{N_X^2}{\alpha_X} = \frac{6}{\alpha_\psi} + \frac{2}{3\alpha_4 + 2\alpha_{2R}}. \tag{14}$$

The boundary condition in Eq.14 applies at the symmetry breaking scale M_{GUT}.

From Eq.8 one finds the following relation between the hypercharge gauge coupling constant g_Y and the $SU(4)$ and $SU(2)_R$ gauge coupling constants g_4 and g_{2R} respectively:

$$\frac{1}{\alpha_Y} = \frac{1}{\alpha_{2R}} + \frac{1}{\frac{3}{2}\alpha_4} \tag{15}$$

where $\alpha_Y \equiv \frac{g_Y^2}{4\pi}$, $\alpha_{2R} \equiv \frac{g_{2R}^2}{4\pi}$ and $\alpha_4 \equiv \frac{g_4^2}{4\pi}$. In terms of the "GUT" (in this case E_6) normalized hypercharge generator $T_Y = \sqrt{\frac{5}{3}}Y$, the coupling constant is $g_1 \equiv \sqrt{\frac{3}{5}}g_Y$:

$$\frac{5}{\alpha_1} = \frac{3}{\alpha_{2R}} + \frac{2}{\alpha_4} \tag{16}$$

where $\alpha_1 \equiv \frac{g_1^2}{4\pi}$. Eq.16 is a boundary condition for the gauge couplings at the Pati-Salam symmetry breaking scale, in this case M_{GUT}. Due to the left-right symmetry D_{LR}, at the G_{4221} symmetry breaking scale we have the additional boundary condition $\alpha_{2L} = \alpha_{2R}$.

In Table 1 we list the values that the generators Y, T_{B-L}, T_R^3, T_ψ and $T_\psi + T_R^3$ (and therefore X) take for the G_{3211} representations of the **27** multiplet. Note that both

$T_\psi + T_R^3$ and Y are zero for ν^c and therefore neither B_Y or B_X couple to right-handed neutrinos. This is a consequence of Goldstone's theorem since the right-handed neutrino comes from the same G_{4221} representation as the Higgs boson component that gets a VEV to break the symmetry.

The $U(1)_X$ associated with the preserved generator in Eq.9 is an anomaly-free gauge group which plays the same role in solving the μ problem as the $U(1)_N$ of the E$_6$SSM, since it allows the coupling Sh_uh_d that generates an effective μ term, while forbidding S^3 and the $\mu h_u h_d$. $U(1)_X$ is broken by the S singlet VEV near the TeV scale, yielding a physical Z' which may be observed at the LHC. We emphasize that this Z' is distinct from those usually considered in the literature based on linear combinations of the E_6 subgroups $U(1)_\psi$ and $U(1)_\chi$ since, in the ME$_6$SSM, $U(1)_\chi$ is necessarily broken at M_p. In particular the Z' of the ME$_6$SSM based on $U(1)_X$ and that of the E$_6$SSM based on $U(1)_N$ will have different physical properties, which we will explore later.

3 Two-Loop RGEs Analysis

In this section we perform a SUSY two-loop RG analysis of the gauge coupling constants, corresponding to the pattern of symmetry breaking discussed in the previous section. According to our assumptions there are three complete **27** SUSY representations of the gauge group E_6 in the spectrum which survive down to low energies, but, unlike the original E$_6$SSM, there are no additional $H', \overline{H}'$ states and so the gauge couplings are not expected to converge at M_{GUT}. We therefore envisage the pattern of symmetry breaking shown in Eq.1. Above the G_{4221} symmetry breaking scale M_{GUT} we assume, in addition to the three **27** representations, some G_{4221} symmetry breaking Higgs sector.

Although a Higgs sector consisting of H_R and $\overline{H}_R$ is perfectly adequate for breaking the G_{4221} symmetry, it does not satisfy D_{LR}. We must therefore also consider an extended Higgs sector including their left-right symmetric partners. A minimal left-right symmetric Higgs sector capable of breaking the G_{4221} symmetry consists of the $SO(10) \otimes U(1)_\psi$ Higgs states $(\mathbf{16}_H)_{\frac{1}{2}}$ and $(\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$, where $(\mathbf{16}_H)_{\frac{1}{2}} = (4,2,1)_{\frac{1}{2}} + (\overline{4},1,\overline{2})_{\frac{1}{2}}$ and $(\overline{\mathbf{16}}_H)_{-\frac{1}{2}} = (\overline{4},\overline{2},1)_{-\frac{1}{2}} + (4,1,2)_{-\frac{1}{2}}$, where the components which get VEVs are $\overline{H}_R = (\overline{4},1,\overline{2})_{\frac{1}{2}}$ and $H_R = (4,1,2)_{-\frac{1}{2}}$. If complete E_6 multiplets are demanded in the entire theory below M_p, then the Pati-Salam breaking Higgs sector at M_{GUT} may be assumed to be $\mathbf{27}_H$ and $\overline{\mathbf{27}}_H$. Therefore, in our analysis we shall consider two possible Higgs sectors which contribute to the SUSY beta functions in the region between M_{GUT} and M_p, namely either $(\mathbf{16}_H)_{\frac{1}{2}} + (\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$ or $\mathbf{27}_H + \overline{\mathbf{27}}_H$, where it is understood that only the G_{4221} gauge group exists in this region, and these Higgs representations must be decomposed under the G_{4221} gauge group.

We therefore investigate the running of the gauge coupling constants at two-loops for an E_6 theory that contains three complete **27** multiplets at low energies and either a $(\mathbf{16}_H)_{\frac{1}{2}} + (\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$ or a $\mathbf{27}_H + \overline{\mathbf{27}}_H$ above the G_{4221} symmetry breaking scale. The E_6 symmetry is assumed to be broken to a left-right symmetric G_{4221} gauge symmetry which is then broken to the standard model gauge group and a $U(1)_X$ group as described

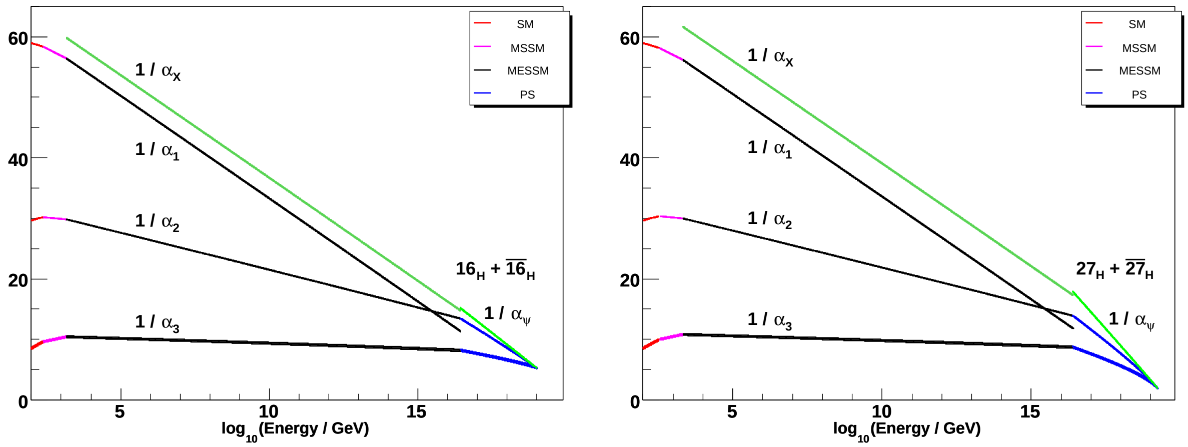

Figure 1: This figure illustrates the two-loop RGEs running of the gauge coupling constants of two different E_6 GUTs with an intermediate left-right symmetric G_{4221} symmetry. Both models contain three $\mathbf{27}$ supermultiplets of E_6 at low energies which contain all the MSSM states as well as new (non-MSSM) states. For the E_6 model in the left-panel we assume effective MSSM and non-MSSM thresholds of 250 GeV and 1.5 TeV respectively. For the E_6 model in the right-panel we assume slightly larger effective MSSM and non-MSSM thresholds of 350 GeV and 2.1 TeV respectively. Above the G_{4221} symmetry breaking scale we include the effects of $SO(10)\otimes U(1)_\psi$ supermultiplets $(\mathbf{16}_H)_{\frac{1}{2}}+(\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$ or E_6 supermultiplets $\mathbf{27}_H+\overline{\mathbf{27}}_H$ for the left-panel and right-panel respectively. From the two-loop RGE analysis we find that the left-right symmetric G_{4221} symmetry is broken at $10^{16.45(3)}$ GeV or $10^{16.40(3)}$ GeV and that gauge coupling unification occurs at $10^{18.95(8)}$ GeV or $10^{19.10(10)}$ GeV for the left-panel and right-panel respectively. For both panels we calculate that $c_{12}^2\equiv\cos\theta_{12}$ is equal to 0.71 to two significant figures where $\tan\theta_{12}\equiv g_{2R}/g_{B-L}$ and the gauge coupling constants are evaluated at the G_{4221} symmetry breaking scale. The $U(1)_X$ charge is given by X/N_X where $X=(T_R^3+T_\psi)-c_{12}^2Y$ and $N_X^2=7-2c_{12}^2+\frac{5}{3}c_{12}^4$.

in section 2. In the previous section we discussed the relation in Eq.16 between the hypercharge and Pati-Salam gauge coupling constants at the G_{4221} symmetry breaking scale. This can be turned into a boundary condition involving purely Standard Model gauge couplings constants at the G_{4221} breaking scale, since $SU(3)_c$ comes from $SU(4)$ so $\alpha_3=\alpha_4$ at this scale, and, as remarked, the D_{LR} symmetry requires that $\alpha_{2R}=\alpha_{2L}$ at the G_{4221} symmetry breaking scale. Therefore Eq.(16) can be re-expressed as:

$$\frac{5}{\alpha_1}=\frac{3}{\alpha_{2L}}+\frac{2}{\alpha_3} \tag{17}$$

Having specified the low energy matter content and thresholds, Eq.17 allows a unique determination of the Pati-Salam breaking scale, by running up the gauge couplings until the condition in Eq.17 is satisfied. In practice we determine the Pati-Salam symmetry breaking scale to be close to the conventional GUT energy scale, and this justifies our use of the notation M_{GUT} to denote the Pati-Salam breaking scale. Above the scale M_{GUT} we run up the two Pati-Salam gauge couplings, namely α_4 and $\alpha_{2L}=\alpha_{2R}$, including, in addition to the three SUSY $\mathbf{27}$ matter representations, also a Pati-Salam SUSY Higgs breaking sector consisting of either $(\mathbf{16}_H)_{\frac{1}{2}}+(\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$ or $\mathbf{27}_H+\overline{\mathbf{27}}_H$. The values of the gauge coupling constants meet at a high energy scale close to the Planck scale $M_p=1.2\times10^{19}$ GeV, which suggests a Planck scale unification of all forces with

	Q	L	u^c	d^c	e^c	ν^c	h_2	h_1	D	$\overline{D}$	S
Y	$\frac{1}{6}$	-$\frac{1}{2}$	-$\frac{2}{3}$	$\frac{1}{3}$	1	0	$\frac{1}{2}$	-$\frac{1}{2}$	-$\frac{1}{3}$	$\frac{1}{3}$	0
X	$\frac{8}{21}$	$\frac{6}{7}$	$\frac{10}{21}$	$\frac{16}{21}$	$\frac{2}{7}$	0	-$\frac{6}{7}$	-$\frac{8}{7}$	-$\frac{16}{21}$	-$\frac{26}{21}$	2
N	1	2	1	2	1	0	-2	-3	-2	-3	5
T_Y	0.129	-0.387	-0.516	0.258	0.775	0	0.387	-0.387	-0.258	0.258	0
T_X	0.150	0.338	0.188	0.301	0.113	0	-0.338	-0.451	-0.301	-0.489	0.789
T_N	0.158	0.316	0.158	0.316	0.158	0	-0.316	-0.474	-0.316	-0.474	0.791

Table 2: This table lists the values that the charges Y, X and N take for the all the G_{3211} representations of the E_6 **27** multiplet. Y is hypercharge, X is the charge of $U(1)_X$ for the model presented in sections 3 and 4 and N is the charge associated with the $U(1)_N$ group in the E$_6$SSM. These charges are normalized by the E_6 normalization constants $N_Y^2 \equiv \frac{5}{3}$, $N_X^2 = 6\frac{62}{147}$ and $N_N^2 \equiv 40$ so that the E_6 normalized charges of the $U(1)_Y$, $U(1)_X$ and $U(1)_N$ groups are given by $T_Y \equiv Y/N_Y$, $T_X \equiv X/N_X$ and $T_N \equiv N/N_N$ respectively, which are also given in the table. N_X and X have been calculated for the case when the mixing angle $\tan\theta_{12} = g_{2R}/g_{B-L}$ is given by $\cos^2\theta_{12} \equiv c_{12}^2 = 5/7$ which, to two significant figures, agrees with the two-loop RGEs analysis presented for the two E_6 theories in section 3.

gravity. Assuming full gauge unification at the high energy scale, the $U(1)_\psi$ gauge coupling is then run down, along with the Pati-Salam gauge couplings, and the $U(1)_X$ gauge coupling is determined at the G_{4221} symmetry breaking scale M_{GUT} from the boundary condition in Eq.13. The low energy gauge couplings, including g_X, are then run up and the procedure is repeated until self-consistent unification is achieved.

The results are shown in Figure 1. For the E_6 theory that contains the $(\mathbf{16}_H)_{\frac{1}{2}} + (\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$ particles, we take a low energy effective threshold of 250 GeV for the MSSM states and therefore an effective threshold of $(6 \times 250) = 1.5$ TeV for the rest of states of the three complete **27** multiplets as assumed in [9], which follows the analysis of effective MSSM thresholds from [8]. For the E_6 theory that contains the $\mathbf{27}_H + \overline{\mathbf{27}}_H$ particles, the MSSM threshold must be increased to 350 GeV (and hence the 1.5 TeV threshold is increased to 2.1 TeV) to ensure unification for the gauge coupling constants of the G_{4221} symmetry. We run the gauge couplings up from low energies to high energies, using as input the SM gauge coupling constants measured on the Z-pole at LEP, which are as follows [14]: $\alpha_1(M_Z) = 0.016947(6)$, $\alpha_2(M_Z) = 0.033813(27)$ and $\alpha_3(M_Z) = 0.1187(20)$. The general two-loop beta functions used to run the gauge couplings can be found in [15]. Using a two-loop renormalization group analysis, we calculate that the G_{4221} symmetry is broken at $10^{16.45(3)}$ GeV or $10^{16.40(3)}$ GeV and that gauge coupling unification occurs at $10^{18.95(8)}$ GeV or $10^{19.10(10)}$ GeV for the E_6 theories that contain $(\mathbf{16}_H)_{\frac{1}{2}} + (\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$ or $\mathbf{27}_H + \overline{\mathbf{27}}_H$ respectively. In [9] the same two-loop RGE analysis was carried out for the E_6 model but without the inclusion of the $U(1)_\psi$ and $U(1)_X$ groups. In terms of a logarithmic scale, the Pati-Salam symmetry breaking scale and unification scale have not been significantly changed from [9] by the inclusion of $U(1)_X$ and $U(1)_\psi$; and Planck scale unification and GUT scale Pati-Salam symmetry breaking is still a possibility.

The value of the gauge coupling constant at the unification scales $10^{18.95(8)}$ GeV or $10^{19.10(10)}$ GeV is $\alpha_P = 0.183(10)$ or $\alpha_P = 0.432(121)$ for the $(\mathbf{16}_H)_{\frac{1}{2}} + (\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$ or

$\mathbf{27}_H + \overline{\mathbf{27}}_H$ particle spectra, respectively. The values of the unified gauge coupling at the Planck scale are much larger than the conventional values of α_{GUT} and indeed are larger even than $\alpha_3(M_Z)$, however they are still in the perturbative regime. Of course there are expected to be large threshold corrections coming from Planck scale physics which are not included in our analysis. Indeed, we would expect that QFT breaks down as we approach the Planck scale, so that our RG analysis ceases to be valid as we approach the Planck scale, as remarked in the Introduction. The precise energy scale E_{new} at which quantum field theories of gravity are expected to break down and new physics takes over is discussed in [16] based on estimates of the scale of violation of (tree-level) unitarity. An upper bound for this new physics energy scale is given by $E^2_{new} = 20[G(\frac{2}{3}N_s + N_f + 4N_V)]^{-1}$ where N_s, N_f and N_V are the number of scalars, fermions and vectors respectively that gravity couples to. Assuming three low-energy $\mathbf{27}$ multiplets, E_{new} would be equal to $10^{18.6}$ GeV which sets an upper bound for the scale at which our quantum field theory analysis (and with any corrections from effective quantum gravity theories included) can no longer be trusted. We have shown that the gauge coupling constants are predicted to be very close to one another at this scale and that, if extrapolated, unify just below M_p. We have naively extrapolated the RGEs up to M_p, even though new physics associated with quantum gravity must enter an order of magnitude below this. The fact that the two PS couplings are very close to each other at E_{new}, and are on a convergent trajectory must be regarded, at best, as a suggestive hint of a unification of the gauge fields with gravity in this approach.

From the previous section, the charge of the $U(1)_X$ group T_X depends on the values that the g_4 and g_{2R} coupling constants take at the G_{4221} symmetry breaking scale, which is written into the cosine c_{12} of the mixing angle $\tan\theta_{12} \equiv g_{2R}/g_{B-L}$. The value that g_4 and g_{2R} take at this scale is automatically set by the unification of the G_{4221} gauge coupling constants. We calculate that, with either $(\mathbf{16}_H)_{\frac{1}{2}} + (\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$ or $\mathbf{27}_H + \overline{\mathbf{27}}_H$ included above the G_{4221} symmetry breaking scale, c^2_{12} is equal 0.71 to two significant figures. However, for convenience we take the physical value of c^2_{12} to be equal to $\frac{5}{7}$ (~ 0.71) so that T_X can be written in terms of fractions. Using this value of c^2_{12} in equation Eq.28, we can calculate T_X for all the standard model particles of the three low-energy $\mathbf{27}$ multiplets. The values that T_X, T_Y and T_N take for the particles of the $\mathbf{27}$ multiplets are given in Table 2, where T_N is the generator associated with the $U(1)_N$ group in the E$_6$SSM.

4 Phenomenology of the new Z' in the ME$_6$SSM

In this section we investigate certain phenological implications of the Z' gauge boson in the ME$_6$SSM. We compare the results to those calculated for the Z' in the E$_6$SSM to see if a possible distinction could be made between the two models in future experiments. We start by reviewing the covariant derivatives for the E$_6$SSM and ME$_6$SSM symmetries below the GUT scale and compare the different $U(1)'$ groups from the two models. The mixing between the Z' of the ME$_6$SSM and the Standard Model Z gauge boson is then calculated and shown to be negligible as in the E$_6$SSM. We then calculate the axial and

vector couplings of the Z' to the low energy particle spectrum and show that the charged lepton vector couplings do differ in the E$_6$SSM and ME$_6$SSM, which could potentially lead to a distinction between the two models in future experiments.

4.1 The Z' of the E$_6$SSM

In the E$_6$SSM the E_6 symmetry is not broken through a Pati-Salam intermediate symmetry but instead breaks to $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_N$ via a $E_6 \to SO(10) \otimes U(1)_\psi \to SU(5) \otimes U(1)_\chi \otimes U(1)_\psi$ symmetry breaking chain. The covariant derivative for the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_N$ symmetry can be written as:

$$D_\mu = \partial_\mu + ig_3 T^n_{3c} A^n_{3c\mu} + ig_{2L} T^s_L A^s_{L\mu} + ig_1 T_Y B_{Y\mu} + ig_N T_N B_{N\mu} \tag{18}$$

where $n = 1 \ldots 8$ and $s = 1 \ldots 3$; $A^n_{3c\mu}$, $A^s_{L\mu}$, $B_{Y\mu}$ and $B_{N\mu}$ are the $SU(3)_c$, $SU(2)_L$, $U(1)_Y$ and $U(1)_N$ quantum fields respectively; g_3, g_{2L}, g_1 and g_N denote the universal gauge coupling constants of the respective fields and T^n_{3c}, T^s_L, T_Y and T_N represent their generators. At low energies the $U(1)_N$ gauge group will be broken, giving rise to a massive Z' gauge boson associated with the E$_6$SSM.

The g_N gauge coupling constant is equal to g_1 to an excellent approximation [6], independent of the energy scale of interest. This is to be compared to the universal gauge coupling constant g_X of the group $U(1)_X$ in the models presented in this section, which is always less than g_1.

Similar to T_Y and T_X, we can split T_N into an E_6 normalization constant N_N and a non-normalized charge N so that $T_N \equiv N/N_N$ where the conventional choice is $N_N^2 \equiv 40$ and $N \equiv \frac{1}{4}\chi + \frac{5}{2}T_\psi$ where $\chi \equiv 2\sqrt{10}T_\chi$ [6].

4.2 The Z' of the ME$_6$SSM

The covariant derivative of the G_{4221} symmetry is discussed in the Appendix and is given by Eq.21 as:

$$D_\mu = \partial_\mu + ig_4 T^m_4 A^m_{4\mu} + ig_{2L} T^s_L A^s_{L\mu} + ig_{2R} T^r_R A^r_{R\mu} + \frac{1}{\sqrt{6}} ig_\psi T_\psi A_{\psi\mu}$$

where $m = 1 \ldots 15$ and $r, s = 1 \ldots 3$; $A^m_{4\mu}$, $A^r_{R\mu}$ and $A_{\psi\mu}$ are the $SU(4)$, $SU(2)_R$ and $U(1)_\psi$ quantum fields respectively; g_4, g_{2R} and g_ψ denote the universal gauge coupling constants of the respective fields; and T^m_4, T^r_R and T_ψ represent their generators.

The covariant derivative of the G_{3211} symmetry is derived in the Appendix and is given by Eq.32 as:

$$D_\mu = \partial_\mu + ig_3 T^n_{3c} A^n_{3c\mu} + ig_{2L} T^s_L A^s_{L\mu} + ig_1 T_Y B_{Y\mu} + ig_X T_X B_{X\mu} \tag{19}$$

where $n = 1\dots8$ and $s = 1\dots3$; and $B_{X\mu}$ and T_X are the gauge field of the $U(1)_X$ group and its (E_6 normalized) charge respectively. At low energies the $U(1)_X$ gauge group will be broken, giving rise to a massive Z' gauge boson associated with the ME$_6$SSM.

As is clear from Table 2, for $c_{12}^2 = \frac{5}{7}$, the T_X and T_N charges are different for all of the G_{3211} representations of the **27** multiplets. However, in the limit $c_{12}^2 = \frac{3}{5}$, corresponding to $g_{2R} = g_4$ at the G_{4221} symmetry breaking scale, then T_X and T_N are identical.[6] This can be seen if one sets $g_{2R} = g_4 = \sqrt{\frac{2}{3}}g_{B-L}$ in Eq.9 and Eq.12, in which case T_X is given by:

$$
\begin{aligned}
T_X &= \frac{1}{4}\left[\frac{4}{2\sqrt{10}}\Big(T_R^3 - \frac{3}{2}T_{B-L}\Big) + \sqrt{15}\Big(T_\psi/\sqrt{6}\Big)\right] \\
&\equiv \frac{1}{4}\left[T_\chi + \sqrt{15}\Big(T_\psi/\sqrt{6}\Big)\right] \\
&\equiv T_\chi \cos\theta + (T_\psi/\sqrt{6})\sin\theta \\
&\equiv T_N \quad \text{(see [14])}
\end{aligned}
$$

where $\theta = \arctan\sqrt{15}$ and T_χ is the E_6 normalized charge for the $U(1)_\chi$ group, which is defined by $SO(10) \to SU(5) \otimes U(1)_\chi$ [13] [7]. In the E$_6$SSM the $U(1)_N$ group is defined as the linear combination of the two groups $U(1)_\chi$ and $U(1)_\psi$ for which the right-handed neutrino is a singlet of the symmetry [6]. This linear combination is $U(1)_\chi \cos\theta + U(1)_\psi \sin\theta$, where $\theta = \arctan\sqrt{15}$ [6], which is the same linear combination of $U(1)_\chi$ and $U(1)_\psi$ that $U(1)_X$ becomes if $g_R = g_4$ as shown above. Thus if $g_R = g_4$ at the G_{4221} symmetry breaking scale, then the covariant derivative for the E$_6$SSM, Eq.18, and the covariant derivative for G_{3211}, Eq.19, become equivalent because of the reasons stated above. However, in the E_6 theories that we have proposed, $c_{12}^2 \sim \frac{5}{7}$ not $\frac{3}{5}$ so that, in general, one expects $g_R \neq g_4$ at the G_{4221} symmetry breaking scale in realistic models.

It is clearly of interest to try to distinguish the Z' of the E$_6$SSM from that of the ME$_6$SSM, since the former one is associated with GUT scale unification, while the latter is associated with Planck scale unification. In the remainder of this section we discuss the phenomenology of the new Z' of the ME$_6$SSM, comparing it to that of the E$_6$SSM. In principle different Z' gauge bosons can be distinguished at the LHC by measuring the leptonic forward-backward charge asymmetries as discussed in [17], providing the mass of the Z' is not much larger than about 2 TeV.

[6] Although T_X and T_N are identical for $c_{12}^2 = 3/5$, X and N and hence N_X and N_N are not. However, we could have defined X and N_X differently so that they agree with N and N_N when $c_{12}^2 = 3/5$.

[7] When $g_{2L} = g_{2R} = g_4$ the Pati-Salam generators can be thought of as $SO(10)$ generators, on the same footing as the $SU(5)$ and $U(1)_\chi$ generators when their gauge couplings are equal, as in the E$_6$SSM. In this limit the above argument shows that there is no distinction between $U(1)_N$ and $U(1)_X$.

4.3 Mixing between Z and the Z' of the ME$_6$SSM

In this section we investigate the mixing between the Z gauge boson and the Z' gauge boson of $U(1)_X$ which is generated once the MSSM Higgs doublets get vacuum expectation values and break the electroweak symmetry. When the MSSM singlet particle S from the low-energy **27** multiplets of the ME$_6$SSM gets a VEV, the $U(1)_X$ group will be broken and a heavy Z' gauge boson will be produced. Then, when h_u and h_d get VEVs, the $SU(2)_L \otimes U(1)_Y$ symmetry will be broken to $U(1)_{em}$ and a heavy neutral Z gauge boson, which is the following mixture of the $SU(2)_L$ and $U(1)_Y$ fields: $Z_\mu = W^3_\mu \cos\theta_W - A_Y \sin\theta_W$ where θ_W is the Electroweak (EW) symmetry mixing angle. Since h_u and h_d transform under $U(1)_X$, they couple to Z' and so mix the Z' and Z gauge bosons when they get VEVs. After S, h_u and h_d get VEVs the mass squared mixing matrix for the Z and Z' gauge bosons is given by [18, 21]:

$$M^2_{ZZ'} = \begin{pmatrix} M^2_Z & \delta M^2 \\ \delta M^2 & M^2_{Z'} \end{pmatrix}$$

where:

$$M^2_Z = (g^2_{2L} + g^2_Y)(Y^h)^2 v^2_h$$

$$M^2_{Z'} = g^2_X v^2_h[(T^{h_1}_X)^2\cos^2\beta + (T^{h_2}_X)^2\sin^2\beta] + g^2_X (T^S_X)^2 s^2$$

$$\delta M^2 = \sqrt{g^2_{2L} + g^2_Y}\; g_X \; Y^h (T^{h_1}_X \cos^2\beta - T^{h_2}_X \sin^2\beta) v^2_h$$

where Y^h is the magnitude of the h_u and h_d Higgs bosons' hypercharge; $T^{h_1}_X$, $T^{h_2}_X$ and T^S_X are the values that the E_6 normalized $U(1)_X$ charge, T_X, takes for the h_1, h_2 and S states respectively; g_{2L} and g_Y are the $SU(2)_L$ and (non-GUT normalized) hypercharge gauge coupling constants evaluated at the EW symmetry breaking scale;[8] g_X is the $U(1)_X$ gauge coupling constant evaluated at the $U(1)_X$ symmetry breaking scale; s is the VEV of the MSSM singlet S; $v_h = \sqrt{v^2_u + v^2_d}$ and $\tan\beta = \frac{v_d}{v_u}$ where v_u and v_d are the vacuum expectation values for the h_u and h_d MSSM Higgs bosons respectively.

The mass eigenstates generated by this mass mixing matrix are:

$$Z_1 = Z\cos\theta_{ZZ'} + Z'\sin\theta_{ZZ'}$$

$$Z_2 = -Z\sin\theta_{ZZ'} + Z'\cos\theta_{ZZ'}$$

with masses $M^2_{Z_1,Z_2} = \frac{1}{2}[M^2_Z + M^2_{Z'} \mp \sqrt{(M^2_Z - M^2_{Z'}) + 4\delta M^4}]$ respectively. The mixing angle $\theta_{ZZ'}$ is given by:

$$\tan(2\theta_{ZZ'}) = \frac{2\delta M^2}{M^2_{Z'} - M^2_Z}.$$

[8] The non-GUT normalized hypercharge coupling constant g_Y is identified as $g_Y \equiv \sqrt{\frac{3}{5}} g_1$.

In terms of this mixing angle the covariant derivative for the mass eigenstate gauge bosons Z_1 and Z_2 is:

$$D_\mu = \partial_\mu + i\left(\frac{\cos\theta_{ZZ'}}{\sqrt{g_Y^2+g_{2L}^2}}(g_{2L}^2 T_L^3 - g_Y^2 Y) - g_X T_X \sin\theta_{ZZ'}\right) Z_{1\mu}$$

$$+ i\left(g_X T_X \cos\theta_{ZZ'} + \frac{\sin\theta_{ZZ'}}{\sqrt{g_Y^2+g_{2L}^2}}(g_{2L}^2 - g_Y^2 Y)\right) Z_{2\mu}$$

where g_Y and g_{2L} are evaluated at the EW symmetry breaking scale and g_X is evaluated at the scale at which S gets a VEV to break the $U(1)_X$ symmetry. Phenomenology constrains the mixing angle $\theta_{ZZ'}$ to be typically less than $2-3\times10^{-3}$ [19] and the mass of the extra neutral gauge boson to be heavier than $500-600$ GeV [20]. We calculate that, if the S particle gets a VEV at 1.5 TeV in the ME$_6$SSM, then $\theta_{ZZ'} = 3\times10^{-3}$ and $M_{Z'} = 544$ GeV so that phenomenologically acceptable values are therefore produced for $s > 1.5$ TeV. This vacuum expectation value is consistent with the RGEs analysis in section 3 and the scale of electroweak symmetry breaking.

Since the mixing angle $\theta_{ZZ'}$ is very small in the ME$_6$SSM, we approximate the two mass eigenstate gauge bosons to be just Z and Z', which are the neutral gauge bosons of the broken $SU(2)_L\otimes U(1)_Y$ and $U(1)_X$ symmetries respectively. The above covariant derivative is then simplified to:

$$D_\mu = \partial_\mu + i\frac{1}{\sqrt{g_Y^2+g_{2L}^2}} Z_\mu\big(g_{2L}^2 T_L^3 - g_Y^2 Y\big) + i g_X Z'_\mu T_X.$$

4.4 Axial and Vector Couplings for Z' in the ME$_6$SSM

If we ignore the mixing between the Z and Z' gauge bosons, then the most general Lagrangian for the $U(1)_X$ group is [21, 22]:

$$\mathcal{L}_X = \frac{1}{2}M_{Z'} Z'^\mu Z'_\mu - \frac{g_X}{2}\sum_i \overline{\psi}_i\gamma^\mu(f_V^i - f_A^i\gamma^5)\psi_i Z'_\mu - \frac{1}{4}F'^{\mu\nu}F'_{\mu\nu} - \frac{\sin\chi}{2}F'^{\mu\nu}F_{\mu\nu}$$

where $F'^{\mu\nu}$ and $F^{\mu\nu}$ are the field strength tensors for $U(1)_X$ and $U(1)_Y$ respectively; ψ_i are the chiral fermions and f_V^i and f_A^i are their vector and axial charges which are given by $f_V^i \equiv \frac{1}{N_X}(X_L^i + X_R^i)$ and $f_A^i \equiv \frac{1}{N_X}(X_L^i - X_R^i)$ where X_L and X_R are the X charges for the left-handed and right-handed particles respectively.

The $\frac{\sin\chi}{2}F'^{\mu\nu}F_{\mu\nu}$ term in the above Lagrangian represents the kinetic term mixing for the two Abelian symmetries $U(1)_Y$ and $U(1)_X$. In general, the kinetic term mixing of two Abelian gauge groups is non-zero because the field strength tensor is gauge-invariant for an Abelian theory. However, if both Abelian groups come from a simple gauge group, such as E_6, then $\sin\chi$ is equal to zero at the tree-level, although non-zero elements could arise at higher orders if the trace of the $U(1)$ charges is not equal to zero for the states

	u	d	e	ν	D	h	S
f_V/N_X	$\frac{1}{2}-\frac{5}{6}c_{12}^2$	$-\frac{1}{2}+\frac{1}{6}c_{12}^2$	$-\frac{1}{2}+\frac{3}{2}c_{12}^2$	$\frac{1}{2}+\frac{1}{2}c_{12}^2$	$\frac{2}{3}c_{12}^2$	$-1+c_{12}^2$	2
f_A/N_X	$\frac{1}{2}+\frac{1}{2}c_{12}^2$	$\frac{3}{2}-\frac{1}{2}c_{12}^2$	$\frac{3}{2}-\frac{1}{2}c_{12}^2$	$\frac{1}{2}+\frac{1}{2}c_{12}^2$	-2	-2	2
f_V	-0.0376	-0.1503	0.2255	0.3382	0.1879	-0.1127	0.7892
f_A	0.3382	0.4510	0.4510	0.3382	-0.7892	-0.7892	0.7892
f_V^0	0.0278	-0.1637	0.1081	0.2996	0.1359	-0.1915	0.7906
f_A^0	0.2996	0.4910	0.4910	0.2996	-0.7906	-0.7906	0.7906

Table 3: In this table we list the axial f_A and vector f_V $U(1)_X$ charge assignments for the G_{3211} representations of the complete **27** E_6 multiplet in the ME$_6$SSM. The assignments for a general ME$_6$SSM model and for the model presented in sections 3 and 5, which has $c_{12}^2 = 5/7$, are both given. The E_6 normalization factor N_X is given by $N_X^2 = 7 - 2c_{12}^2 + \frac{5}{3}c_{12}^4$ for a general model and is equal to $6\frac{62}{147}$ when $c_{12}^2 = 5/7$. We have also included the axial and vector $U(1)_N$ charge assignments f_V^0 and f_A^0 in the E$_6$SSM so that a comparison can be made to the corresponding ME$_6$SSM quantities f_V and f_A.

lighter than the energy scale of interest [22]. The trace of the $U(1)_Y$ and $U(1)_X$ charges is given by:

$$\text{Tr }(T_Y\ T_X) = \sum_{\text{i=chiral fields}} (T_Y^i\ T_X^i).$$

This trace is only non-zero if incomplete GUT multiplets are present in the low energy particle spectrum. There are no low-energy incomplete E_6 multiplets in the model we presented in section 4 and so $\sin\chi = 0$ at the tree-level and at higher orders in this particular case. There is therefore no kinetic term mixing between the $U(1)_Y$ and $U(1)_X$ groups in the ME$_6$SSM. In the E$_6$SSM two additional EW doublets from incomplete E_6 multiplets are kept light so that, in this case, $\sin\chi$ is non-zero [6].

The second term in the $U(1)_X$ Lagrangian $\mathcal{L}_X$ represents the interaction between the Z' gauge boson and the fermions. In table 3 we list the vector and axial $U(1)_X$ charges for the G_{4221} representations of the complete **27** low-energy E_6 multiplets in a general E_6 theory and the ME$_6$SSM, which has $c_{12}^2 = 5/7$. We also list the vector and axial $U(1)_N$ charges of the E$_6$SSM for the low-energy **27** multiplets for a comparison. The differences between the values of the vector and axial couplings of the two Z' gauge bosons of the $U(1)_X$ and $U(1)_N$ groups are due to the difference in value between the E_6 normalized T_X and T_N charges and the fact that the kinetic term mixing between the $U(1)_Y$ and the $U(1)'$ groups is non-zero in the E$_6$SSM but zero in the ME$_6$SSM. The largest difference between the vector and axial couplings of $U(1)_X$ and $U(1)_N$ exists for the charged leptons where the vector coupling for $U(1)_X$ is a factor of two larger than for $U(1)_N$.

field	$SU(4)\otimes SU(2)_L\otimes SU(2)_R\otimes U(1)_\psi$	$U(1)_R$	Z_2^H
F_i, F_i^c	$(4,2,1)_{\frac{1}{2}}$, $(\overline{4},1,2)_{\frac{1}{2}}$	1	$-$
h_3, h_α	$(1,2,2)_{-1}$	0	$+,-$
S_3, S_α	$(1,1,1)_2$	2	$+,-$
$\mathcal{D}_i$	$(6,1,1)_{-1}$	0	$-$
M, Σ	$(1,1,1)_0$	0	$+,-$
H_L, $\overline{H}_R$	$(4,2,1)_{\frac{1}{2}}$, $(\overline{4},1,2)_{\frac{1}{2}}$	2	$+$
$\overline{H}_L$, H_R	$(\overline{4},2,1)_{-\frac{1}{2}}$, $(4,1,2)_{-\frac{1}{2}}$	0	$+$

Table 4: This table lists all the charge assignments for the G_{4221} representations of the ME_6SSM, where $i=1\dots 3$ is a family index and $\alpha=1,2$. The $U(1)_R$ is an R-symmetry and Z_2^H distinguishes the third family Higgs which get VEVs. These symmetries obey the G_{4221} symmetry but not the E_6 symmetry since the latter is assumed to be broken by quantum gravity effects. The superpotential terms that are allowed by these symmetries are given in Table 5. The h_3 supermultiplet contains the MSSM Higgs bosons and S_3 is the MSSM singlet that generates an effective μ-term $\lambda_S S_3 h_3 h_3$. The H_L and $\overline{H}_L$ Higgs bosons are required to satisfy the left-right discrete operator D_{LR} that is defined by $E_6\to G_{4221}\otimes D_{LR}$. Σ and M are E_6 singlets that are assumed to get VEVs at 10^{7-11} GeV and $10^{16.4}$ GeV respectively.

5 A Realistic Model

In this section we construct a realistic ME$_6$SSM, focussing on the model building issues. The ME$_6$SSM has a more 'minimal' particle content than the E$_6$SSM since it only contains three complete **27** multiplets at low energies whereas the E$_6$SSM contains two additional EW doublets which can be considered as states of incomplete **27** and $\overline{\mathbf{27}}$ E_6 multiplets. From the RGEs analysis, unification of the G_{4221} gauge coupling constants occurs near the Planck scale where an E_6 symmetry should in principle exist. However, given the expected strength of quantum gravity at this scale, it is likely that any such E_6 symmetry is for all practical purposes broken by gravitational effects. Therefore, the model that we propose in this section is chosen to not respect an E_6 symmetry but instead obey the G_{4221} symmetry that exists between the conventional GUT and Planck scales where quantum gravity effects will not be so significant.

Under $E_6\to SO(10)\otimes U(1)_\psi\to G_{4221}$, the fundamental E_6 representation breaks into the following: $\mathbf{27}\to 16_{\frac{1}{2}}+10_{-1}+1_2\to F+F^c+h+\mathcal{D}+S$ where $F\equiv(4,2,1)_{\frac{1}{2}}$ contains one family of the left-handed quarks and leptons, $F^c\equiv(\overline{4},1,\overline{2})_{\frac{1}{2}}$ can contain one family of the charge-conjugated quarks and leptons, which includes a charge-conjugated neutrino, $h\equiv(1,2,2)_{-1}$ contains the MSSM Higgs doublets h_u and h_d, while $\mathcal{D}\equiv(6,1,1)_{-1}$ contains two colour-triplet weak-singlet particles, and $S\equiv(1,1,1)_2$ is a MSSM singlet. Including three families contained in three $\mathbf{27}_i$ reps, then, without further constraints on the theory, the allowed couplings are contained in the tensor products [6,12]:

$$\mathbf{27}_i\mathbf{27}_j\mathbf{27}_k\to F_iF_j^ch_k+F_iF_j\mathcal{D}_k+F_i^cF_j^c\mathcal{D}_k+S_ih_jh_k+S_i\mathcal{D}_j\mathcal{D}_k \tag{20}$$

where $i,j,k=1\dots 3$ are family indices. However not all these terms are desirable since the presence of extra Higgs doublets can give rise to flavour changing neutral currents

(FCNCs) and the presence of light colour triplets can induce proton decay. Therefore extra symmetries are required to control the couplings, a suitable choice being the R-symmetry and the discrete Z_2^H symmetry displayed in Table 4, which reduces the allowed couplings to those shown in Table 5, where we also display the lowest order non-renormalizable terms. We now discuss the physics of the allowed and suppressed terms.

5.1 Suppressed Flavour Changing Neutral Currents

We take the $F_i F_j^c h_3$ superpotential terms to contain the MSSM Yukawa couplings since we assume that the third generation h_3 gives the MSSM Higgs doublets. The other h_α states are taken to not get VEVs and will cause FCNCs unless the superpotential term $F_i F_j^c h_\alpha$, where $\alpha = 1, 2$, is forbidden or highly suppressed by some new symmetry [6]. Here we forbid these terms using a Z_2^H discrete symmetry that respects the G_{4221} symmetry but not the Planck-scale E_6 symmetry since the latter is assumed to be broken by quantum gravity. Under this Z_2^H symmetry the 'matter particles' F_i and F_i^c and 'non-Higgs' particles h_α are taken to have $Z_2^H = -1$ and the MSSM Higgs doublets from h_3 are assumed to have $Z_2^H = +1$. The FCNC inducing terms $F_i F_j^c h_\alpha$ are therefore forbidden by the Z_2^H symmetry and the MSSM superpotential generating terms $F_i F_j^c h_3$ are allowed.[9]

However, we show later that, although the $F_i F_j^c h_\alpha$ terms are forbidden at the renormalizable level by Z_2^H, they are still generated from non-renormalizable terms, which are heavily suppressed so that the induced FCNCs are not significant. The Z_2^H symmetry used here forbids the FCNCs in the same way that the Z_2^H symmetry of the E_6SSM forbids the FCNCs from the h_α 'non-Higgs' particles in that model [6].

5.2 The μ-Term and Exotic Mass Terms

As with h_i, we assume that only the third generation of the S_i states gets a vacuum expectation value so that the $S_3 h_3 h_3$ term, from the G_{4221} superpotential term $S_i h_j h_k$, will generate an effective MSSM μ-term. For this term to be allowed by the Z_2^H symmetry, the S_3 particles must have $Z_2^H = +1$.

This S_3 particle is also used to give mass to the 'non-Higgs' particles h_α and colour-triplet particles $\mathcal{D}_i$ via the terms $S_3 h_\alpha h_\beta$ and $S_3 \mathcal{D}_i \mathcal{D}_j$ respectively where $\beta = 1, 2$. For general $U(1)'$ models, the $S_3 \mathcal{D}_i \mathcal{D}_j$ superpotential term has been shown to induce a VEV for the singlet S_3 so that it can generate an effective μ-term [24,25]. The Yukawa coupling constant for the $S_3 \mathcal{D}_i \mathcal{D}_j$ term will, in general, contribute to the renormalization group evolution of the soft singlet mass m_S^2 causing it to run negative in the scalar potential. The VEV of S_3 then carries information about soft supersymmetry breaking from the parameter m_S^2. Therefore, the effective μ parameter is now correlated in some way to the SUSY breaking mechanism and its observed correlation with the soft Higgs mass terms

[9] Forbidding or highly suppressing these terms could explain why only h_3 gets a VEV since then the other h_i states won't directly couple to the top Yukawa coupling.

Allowed couplings	Physics
$F_i F_j^c h_3$	MSSM superpotential
$S_3 h_3 h_3$	Effective MSSM μ-term
$S_3 h_\alpha h_\beta$	h_α mass
$S_3 \mathcal{D}_i \mathcal{D}_j$	$\mathcal{D}_i$ mass
$S_\alpha h_\beta h_3$	S_α mass
$\frac{1}{M_p}\Sigma(F_i F_j \mathcal{D}_k + F_i^c F_j^c \mathcal{D}_k)$	Allows $\mathcal{D}_i$ and proton decay
$\frac{1}{M_P}\Sigma F_i F_j^c h_\alpha$	Heavily suppressed FCNCs
$\frac{1}{M_P}\Sigma S_\alpha \mathcal{D}_i \mathcal{D}_j$	Harmless
$\frac{1}{M_P}\Sigma S_\alpha h_\beta h_\gamma$	Harmless
$\frac{1}{M_P}\Sigma S_\alpha h_3 h_3$	Harmless
$\frac{1}{M_p} F_i^c F_j^c H_R H_R$	ν^c mass
$\frac{1}{M_p} F_i F_j \overline{H}_L \overline{H}_L$	Harmless
$M(H_R \overline{H}_R + H_L \overline{H}_L)$	$\mathbf{16}_H + \overline{\mathbf{16}}_H$ mass

Table 5: This table lists the G_{4221} superpotential terms that are obtained from all the renormalizable and first-order non-renormalizable E_6 tensor products of $\mathbf{27}_i$, $(\mathbf{16}_H)_{\frac{1}{2}} + (\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$ (from a $\mathbf{27}+\overline{\mathbf{27}}$), M and Σ that are allowed by the Z_2^H and $U(1)_R$ symmetries of the ME$_6$SSM, as discussed in section 5 and table 4. The indices $i, j, k = 1 \ldots 3$ and $\alpha, \beta, \gamma = 1, 2$ are family indices.

can be understood [24,26]. That is, the μ problem of the MSSM should not exist in this model. We show below that the $S_\alpha \mathcal{D}_i \mathcal{D}_j$ and $S_\alpha h_\beta h_\gamma$ (where $\gamma = 1, 2$) superpotential terms are forbidden at tee-level so that S_α should not acquire VEVs. These S_α particles will instead get mass from the $S_\alpha h_\beta h_3$ superpotential terms where S_α has $Z_2^H = -1$.

5.3 Exotic Decay and Suppressed Proton Decay

The remaining G_{4221} superpotential terms to be discussed from Eq.20 are $F_i F_j \mathcal{D}_k$ and $F_i^c F_j^c \mathcal{D}_k$. These will cause rapid proton decay in this model unless they are highly suppressed or forbidden by some symmetry [4,6]. The Standard Model representations of these superpotential terms are often found to some degree in other GUTs and the rapid proton decay problems are often solved using some doublet-triplet splitting mechanism that gives large (above the GUT scale) mass to the analogue of the $\mathcal{D}_i$ (triplet) particles, but EW mass to the Higgs doublets. However, in our model we do not give a large mass to the $\mathcal{D}_i$ particles because gauge anomalies would then exist, due to the $U(1)_X$ group, and Planck scale unification would be lost. Also, as discussed above, the $\mathcal{D}_i$ particles can be used to help induce a VEV for the S_3 particle, around the EW scale, if they contribute to the low energy theory. We must therefore highly suppress the $F_i F_j \mathcal{D}_k$ and $F_i^c F_j^c \mathcal{D}_k$ superpotential terms using a small Yukawa coupling constant rather than using the general GUT method of creating large $\mathcal{D}_i$ masses.

Note that these superpotential terms must be suppressed rather than forbidden since the $\mathcal{D}_i$ particles would become stable, strongly interacting particles with TeV

scale masses. Such particles cannot exist in nature and in fact could potentially cause problems for nucleosynthesis even if they are unstable with a lifetime greater than just 0.1s [27]. Therefore, the $F_iF_j\mathcal{D}_k$ and $F_i^cF_j^c\mathcal{D}_k$ terms should not be suppressed by too small a Yukawa coupling constant for the lifetime of $\mathcal{D}_i$ to exceed 0.1s, or too large a Yukawa coupling constant for the proton's lifetime to be smaller than the present experimental limits.

To overcome these problems we use the same method that is used in the model presented in [9]. That is, we forbid the $F_iF_j\mathcal{D}_k$ and $F_i^cF_j^c\mathcal{D}_k$ superpotential terms at the tree-level but generate them from the non-renormalizable terms $\Sigma F_iF_j\mathcal{D}_k$ and $\Sigma F_i^cF_j^c\mathcal{D}_k$, where Σ is an E_6 singlet which is assumed to get a VEV at some high energy scale, by taking both both Σ and $\mathcal{D}_i$ to have $Z_2^H = -1$. These non-renormalizable superpotential terms are expected to survive from the Planck scale and so will likely be suppressed by a factor of $1/M_p$. We can therefore control the degree of suppression of the $F_iF_j\mathcal{D}_k$ and $F_i^cF_j^c\mathcal{D}_k$ terms by choosing the energy scale at which Σ gets a VEV. Below we estimate the energy scales at which Σ can get a VEV so that the proton's lifetime is above the experimental limits and the $\mathcal{D}_i$ particles have a lifetime less than 0.1s.

The superpotential terms $\lambda_{ijk}F_iF_j\mathcal{D}_k$ and $\lambda_{ijk}F_i^cF_j^c\mathcal{D}_k$ cause proton decay through the decay channels $p \to K^+\overline{\nu}$ via $d = 5$ operators (through the $S_3\mathcal{D}_i\mathcal{D}_j$ term which is responsible for the triplet mass m_D) and $p \to \pi^0 e^+$ via $d = 6$ operators with matrix elements proportional to $\lambda^2/m_D m_{SUSY}$ and λ^2/m_D^2 respectively [28,29], where m_{SUSY} is the mass scale for the Standard Model's superpartners. The present experimental limits on the proton's lifetime for the $p \to K^+\overline{\nu}$ and $p \to \pi^0 e^+$ decay channels are 1.6×10^{33} years and 5.0×10^{33} years respectively [14]. The mass m_D of the $\mathcal{D}$ particles required to suppress the $p \to K^+\overline{\nu}$ and $p \to \pi^0 e^+$ matrix elements enough for proton decay to not have been observed is given in various papers (see for example [3,30]) where no fine tuning of the Yukawa coupling λ is used. These calculations assume that a doublet-triplet splitting mechanism can be implemented to give large GUT scale masses to the triplet $\mathcal{D}$ particles but EW scale masses to the Standard Model Higgs doublets. Here we instead assume that the mass of $\mathcal{D}$ is not very different from the EW scale (e.g. $m_D = 1.5$ TeV) and that the Yukawa coupling λ is very small compared to the Yukawa couplings of the Standard Model. We make a rough order of magnitude estimate for the value of the Yukawa coupling λ required for unobservable proton decay through the $d = 5$ and $d = 6$ channels, for $m_D = 1.5$ TeV, by scaling the results obtained from [3,30] and using the fact that the matrix elements for $p \to K^+\overline{\nu}$ and $p \to \pi^0 e^+$ are proportional to $\lambda^2/m_D m_{SUSY}$ and λ^2/m_D^2 respectively. In the case of triplets that are much heavier than the doublets, it is the $d = 5$ channel that sets the higher limit on the mass of the triplets and this is usually higher than the GUT scale [3,28]. For $m_D = 1.5$ TeV, however, the $d = 5$ and $d = 6$ decay channels have similar decay rates since M_{SUSY} is expected to be close to the TeV scale and the decay rates depend on the square of the matrix elements. In what follows we shall choose to scale the results based on the $d = 6$ operator, since these turn out to give slightly stronger limits on λ.

According to [3] the triplets $\mathcal{D}$ must have mass larger than 10^{10-11} GeV for the lifetime of the proton to be greater than 5.0×10^{33} years for the non-SUSY $d = 6$ channel $p \to K^+\overline{\nu}$ and for no fine-tuning of the Yukawa coupling constant of the $F_iF_j\mathcal{D}_k$

and $F_i^c F_j^c \mathcal{D}_k$ superpotential terms. Assuming that the Yukawa coupling used in [3] is of order unity, since it is not specified, we use the square of the matrix element of $p \to K^+\overline{\nu}$ to estimate that the Yukawa coupling λ must be less than the following when the $\mathcal{D}$ triplets have mass equal to $m_D = 1.5$ TeV:

$$\lambda^4 \lesssim \frac{1}{5.0 \times 10^{33}\ yrs} \times \left(\frac{1.5 \text{ TeV}}{10^{11-12} \text{ GeV}}\right)^4 \quad \Rightarrow \lambda \lesssim 10^{-8}.$$

As mentioned above, we forbid the $F_i F_j \mathcal{D}_k$ and $F_i^c F_j^c \mathcal{D}_k$ superpotential terms by the Z_2^H symmetry but effectively generate them from the non-renormalizable terms $\frac{1}{M_p}\Sigma F_i F_j \mathcal{D}_k$ and $\frac{1}{M_p}\Sigma F_i^c F_j^c \mathcal{D}_k$ when Σ gets a VEV. To generate an effective Yukawa coupling smaller than 10^{-8} to avoid experimentally observable proton decay, Σ must get a VEV less than 10^{11} GeV.

The effective superpotential terms $F_i F_j \mathcal{D}_k$ and $F_i^c F_j^c \mathcal{D}_k$, generated from $\frac{1}{M_p}\Sigma F_i F_j \mathcal{D}_k$ and $\frac{1}{M_p}\Sigma F_i^c F_j^c \mathcal{D}_k$, are the only source for the $\mathcal{D}_i$ particles to decay. Assuming that $m_{\widetilde{t}} < m_D$, where $m_{\widetilde{t}}$ is the mass of the heaviest stop, the D^c Standard Model representation of the G_{4221} $\mathcal{D}$ particle will predominantly decay through the channel $D^c \to \widetilde{t} + b$ [6]. Using the standard 2-body decay kinematic formula [14] we estimate that the decay rate for $D^c \to \widetilde{t} + b$, under the assumption that $m_b \ll m_{\widetilde{t}}$, is:

$$d\Gamma \sim \frac{1}{32\pi^2}|\mathcal{M}|^2 \frac{m_D^2 - m_{\widetilde{t}}^2}{2m_D^3} d\Omega$$

At tree-level, a rough order of magnitude estimate of the matrix $\mathcal{M}$ for the $D^c \to \widetilde{t}+b$ decay channel gives:

$$|\mathcal{M}|^2 \sim 2(m_D^2 - m_{\widetilde{t}}^2)\lambda^2$$

Taking the mass of the stop to be around the TeV scale, we estimate that the $F_i F_j \mathcal{D}_k$ and $F_i^c F_j^c \mathcal{D}_k$ operators must be multiplied by an effective Yukawa coupling λ that is greater than 10^{-13} for the $\mathcal{D}_i$ particles to have a lifetime less than 0.1s.

If the stop is assumed to be heavier than D^c (e.g. $m_{\widetilde{t}} = 2$ TeV, $m_D = 1.5$ TeV) then the predominant decay channel will most likely be $D^c \to b + c + \chi_0$ where χ_0 is a neutralino [6]. Under the assumption that $m_c \ll m_b \ll m_{\chi_0}$, a tree-level order of magnitude estimate for the decay rate of this channel can be shown to require an effective Yukawa coupling λ about an order of magnitude larger than when the stop is lighter than $\mathcal{D}$. We therefore require that $\lambda \gtrsim 10^{-12}$ for the $\mathcal{D}_i$ particles to have a lifetime less than 0.1s.

The superpotential terms $\lambda_{ijk} F_i F_j \mathcal{D}_k$ and $\lambda_{ijk} F_i^c F_j^c \mathcal{D}_k$ are effectively generated from the Planck-suppressed operators $\frac{1}{M_p}\Sigma F_i F_j \mathcal{D}_k$ and $\frac{1}{M_p}\Sigma F_i^c F_j^c \mathcal{D}_k$ and so the Yukawa coupling λ is given by $< \Sigma > / M_p$ where $< \Sigma >$ is the VEV of the Σ particle. To avoid cosmological difficulties from the $\mathcal{D}_i$ particles, Σ must therefore get a VEV greater than about 10^7 GeV. Therefore, to avoid experimentally observable proton decay and cosmological issues with the $\mathcal{D}_i$ particles, we require that the E_6 singlet Σ should get a VEV between 10^{7-11} GeV.

5.4 R-Symmetry and R-Parity

To ensure that the LSP is stable in this model, so that it is a candidate for dark matter, we derive R-parity from the $U(1)_R$ symmetry [23], which commutes with the G_{4221} symmetry but not the E_6 symmetry because the latter may not be respected by low energy symmetries as it is assumed to be broken by quantum gravity effects. To allow the G_{4221} superpotential terms, which respect the Z_2^H discrete symmetry, and to derive a generalization of the MSSM R-parity, the G_{4221} supermultiplets of the three $\mathbf{27}$ E_6 have the following $U(1)_R$ R-charge assignments: F_i and F_i^c have $R = +1$; h_3, h_α, $\mathcal{D}_i$ and Σ have $R = 0$; and S_3 and S_α have $R = +2$ (see table 4). The 16_H state also has $R = +2$ so that when it gets a VEV the $U(1)_R$ is broken to a Z_2 discrete symmetry, which we call Z_2^R. Under this Z_2^R symmetry the scalar components of F_i, F_i^c and the fermionic components of h_3 (the MSSM sparticles) all have $Z_2^R = -1$ while the fermionic components of F_i and F_i^c and the scalar components of h_3 (the MSSM particles) all have $Z_2^R = +1$. The Z_2^R symmetry is therefore equivalent to the R-parity of the MSSM for the F_i, F_i^c and h_3 supermultiplets. The h_α, $\mathcal{D}_i$, S_i and Σ supermultiplets are not in the MSSM. All the scalar components of these 'new' supermultiplets can be shown to have $Z_2^R = +1$ while all the fermionic components have $Z_2^R = -1$. Therefore F_i and F_i^c are the only supermultiplets in the theory which have $Z_2^R = +1$ for their fermionic components and $Z_2^R = -1$ for their scalar components. This Z_2^R symmetry stops the 'non-MSSM' particles from allowing the MSSM LSP to decay as well as operating as the R-parity of the MSSM. The introduction of the Z_2^R symmetry therefore ensures a stable dark matter candidate, the MSSM LSP.

Note that the Z_2^H symmetry in table 4 is equivalent to an MSSM matter-parity. Therefore if it was left unbroken then it would also prevent the MSSM LSP from decaying. However, as discussed in section 5.3, the Z_2^H symmetry is broken by the E_6 singlet Σ at around 10^{7-11} GeV generating the effective operators $F_iF_j\mathcal{D}_k$, $F_i^cF_j^c\mathcal{D}_k$ and $F_iF_j^ch_\alpha$ that disrespect Z_2^H, and enabling the MSSM LSP to decay. Hence the Z_2^R symmetry must be introduced in addition to the Z_2^H symmetry so that the MSSM LSP is stable.

5.5 $\mathbf{16}_H + \overline{\mathbf{16}}_H$ Mass

In addition to the three $\mathbf{27}$ E_6 multiplets, which are present at low energies, the Higgs states from $(\mathbf{16}_H)_{\frac{1}{2}} + (\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$ or $\mathbf{27}_H + \overline{\mathbf{27}}_H$ must be given a mass at the conventional GUT scale so that the G_{4221} symmetry can be broken and the gauge coupling constants can unify at the Planck scale. Here we only consider a model that contains the $(\mathbf{16}_H)_{\frac{1}{2}} + (\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$ above the GUT scale and briefly mention what changes should be made to such a model if $\mathbf{27}_H + \overline{\mathbf{27}}_H$ are used to break the G_{4221} symmetry instead.

To give the required GUT scale masses to the $(\mathbf{16}_H)_{\frac{1}{2}} + (\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$ multiplets, we introduce an E_6 singlet M that is assumed to get a VEV at this particular energy scale and which couples to the G_{4221} representations of the $(\mathbf{16}_H)_{\frac{1}{2}}$ and $(\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$ multiplets via the superpotential term $M(\mathbf{16}_H)_{\frac{1}{2}}(\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$. We take the $(\mathbf{16}_H)_{\frac{1}{2}}$ supermultiplet to have an R-charge of +2 so that certain phenomenologically problematic operators are forbidden.

Then, to allow the $M(\mathbf{16}_H)_{\frac{1}{2}}(\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$ term, the M and $(\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$ supermultiplets are both given an R-charge of 0.[10] If $\mathbf{27}_H + \overline{\mathbf{27}}_H$ is included between the GUT and Planck scales, rather than $(\mathbf{16}_H)_{\frac{1}{2}} + (\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$, then another Z_2 must be applied to the $\mathbf{27}_H$ and $\overline{\mathbf{27}}_H$ multiplets to forbid certain phenomenologically problematic superpotential terms.

5.6 Neutrino Mass

These R-charge assignments forbid phenomenologically-problematic terms and allow the charge-conjugated neutrinos, from F_i^c, to obtain a large Majorana mass $\mathcal{O}(M_{GUT}^2/M_p)$ from a $\frac{1}{M_p}F_i^c F_j^c(\overline{\mathbf{16}}_H)_{-\frac{1}{2}}(\overline{\mathbf{16}}_H)_{-\frac{1}{2}} \equiv \frac{1}{M_p}F_i^c F_j^c H_R H_R$ superpotential term. This term will create a conventional see-saw mechanism for the left-handed neutrinos when h_3 gets a VEV in the superpotential term $F_i F_j^c h_3$. This and $\frac{1}{M_p}F_i F_j \overline{H}_L \overline{H}_L$, which is phenomenologically harmless, are the only superpotential terms that contain interactions between the three $\mathbf{27}$ E_6 multiplets and the $(\mathbf{16}_H)_{\frac{1}{2}} + (\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$ and M multiplets.

6 Summary and Conclusion

We have proposed a Minimal E_6 Supersymmetric Standard Model (ME$_6$SSM) based on three low energy reducible $\mathbf{27}$ representations of the Standard Model gauge group which has many attractive features compared to the MSSM. In particular it provides a solution to the μ problem and doublet-triplet splitting problem, without re-introducing either of these problems. Above the conventional GUT scale the ME$_6$SSM is embedded into a left-right symmetric Supersymmetric Pati-Salam model, which allows complete gauge unification at the Planck scale, subject to gravitational uncertainties. Although we have not studied it here, it is also clear that fine-tuning in such models will be significantly reduced compared to the MSSM due to the enhanced Higgs mass.

At low energies there is an additional $U(1)_X$ gauge group, consisting of a novel and non-trivial linear combination of one Abelian and two non-Abelian Pati-Salam generators. The $U(1)_X$ is broken at the TeV scale by the same singlet that also generates the effective μ term, resulting in a new low energy Z' gauge boson. We compared the Z' of the ME$_6$SSM (produced via the Pati-Salam breaking chain of E_6, where E_6 is broken at the Planck scale) to the Z' of the E$_6$SSM (from the $SU(5)$ breaking chain of E_6, where E_6 is broken at the GUT scale) and discussed how they can be distinguished by their different couplings. The possible discovery of such Z' gauge bosons is straightforward at the LHC and the different couplings should enable the two models to be resolved experimentally. In particular, the most significant difference between the vector and axial couplings of the Z' of the E$_6$SSM and ME$_6$SSM is in the vector coupling of the charges leptons, which is twice as large in the ME$_6$SSM as in the E$_6$SSM.

We emphasise that the presence of additional threshold corrections at the Planck scale will not change the Pati-Salam breaking scale or the values of the Standard Model

[10]Note that the bilinear term $(\mathbf{16}_H)_{\frac{1}{2}}(\overline{\mathbf{16}}_H)_{-\frac{1}{2}}$ is also allowed by the symmetries of the model. We assume that the dimensional coupling constant for this term is less than or equal to M_{GUT}.

gauge couplings at this scale to one loop order. However, since these quantities are determined by running up the couplings from low energies, there will be some sensitivity to TeV scale threshold corrections. Since the vector and axial vector couplings of the Z' are determined from the values of the Standard Model gauge couplings at the Pati-Salam breaking scale, there will therefore be little sensitivity to Planck scale threshold corrections on the determined vector and axial vector couplings of the Z'.

We have introduced an R-symmetry and discrete Z_2^H symmetry that addresses the potential major phenomenological problems such as flavour changing neutral currents and proton decay, which would otherwise be introduced to the theory by colour triplet fermions and extra non-Higgs doublets from the three copies of the **27** multiplet. In the ME$_6$SSM, right-handed Majorana masses of the correct order of magnitude naturally arise from the Higgs mechanism that breaks the intermediate Pati-Salam and $U(1)_\psi$ symmetry to the standard model and $U(1)_X$ gauge group, leading to a conventional see-saw mechanism. It should be possible to embed the model presented here into a realistic flavour model describing all quark and lepton masses, leading to predictions for the exotic colour triplet non-Higgs fermion masses, which will be the subject of a future study.

In conclusion, the ME$_6$SSM has clear advantages over both the MSSM and NMSSM, and even the E$_6$SSM, which make it a serious candidate SUSY Standard Model. It also has a certain elegance in the way that the low energy theory contains only complete reducible **27** representations that also allow for anomaly cancellation of the gauged $U(1)_X$, which we find quite appealing. We have shown that the potentially dangerous couplings of the exotic particles can readily be tamed by simple symmetries, leading to exciting predictions at the LHC of exotic colour triplet fermions and a new Z' with distinctive couplings. The discovery and study of such new particles could provide a glimpse into the physics of unification at the Planck scale.

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Appendix: Symmetry Breaking G_{4221} to G_{3211}

Since the $U(1)_X$ group does not appear to have been considered in the literature, we illustrate in detail how the H_R and $\overline{H}_R$ Higgs bosons can generate it and also break the $G_{4221} = SU(4) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_\psi$ symmetry to the $G_{3211} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ symmetry.

We start with the covariant derivative of the G_{4221} symmetry, which can be written

as:

$$D_\mu = \partial_\mu + ig_4 T_4^m A_{4\mu}^m + ig_{2L} T_L^s A_{L\mu}^s + ig_{2R} T_R^r A_{R\mu}^r + \frac{1}{\sqrt{6}} ig_\psi T_\psi A_{\psi\mu} \tag{21}$$

where $m = 1 \ldots 15$ and $r, s = 1 \ldots 3$; $A_{4\mu}^m$, $A_{L\mu}^s$, $A_{R\mu}^r$ and $A_{\psi\mu}$ are the $SU(4)_c$, $SU(2)_L$, $SU(2)_R$ and $U(1)_\psi$ quantum fields respectively; g_4, g_{2L}, g_{2R} and g_ψ denote the universal gauge coupling constants of the respective fields and T_4^m, T_L^s, T_R^r and T_ψ represent their generators. All of the T_4^m, T_R^r, T_L^s and T_ψ generators are derived from components of the E_6 generators G^a, which we choose to E_6 normalize, for the fundamental representation **27**, by:

$$\mathrm{Tr}(G^a\ G^b) = 3\delta^{ab} \tag{22}$$

where $a, b = 1 \ldots 78$.

Then, with this normalization, the Pati-Salam generators T_4^m, T_R^r and T_L^s are normalized for the fundamental representations of $SU(4)$, $SU(2)_R$ and $SU(2)_L$ respectively, by:[11]

$$Tr(T_4^m\ T_4^n) = \frac{1}{2}\delta^{mn},$$

$$Tr(T_R^r\ T_R^s) = Tr(T_L^r\ T_L^s) = \frac{1}{2}\delta^{rs}$$

where $m, n = 1 \ldots 15$.

The $U(1)_\psi$ charge $\frac{1}{\sqrt{6}}T_\psi$ is a diagonal E_6 generator, which we choose to be the 78th generator $G^{78} = \frac{1}{\sqrt{6}}T_\psi$, and is therefore normalized by Eq.22 to give:

$$\frac{1}{6}\sum_{\mathbf{27}} T_\psi^2 = 3 \tag{23}$$

where the sum is over all the G_{4221} representations that make up the fundamental **27** multiplet of E_6.

To break G_{4221} to $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ we use the Higgs bosons H_R and $\overline{H}_R$ that transform as $(4, 1, 2)_{-\frac{1}{2}}$ and $(\overline{4}, 1, \overline{2})_{\frac{1}{2}}$ under G_{4221} respectively. These are the smallest G_{4221} multiplets that can be used to break the Pati-Salam symmetry directly to the standard model gauge group. When H_R and $\overline{H}_R$ develop VEVs in the ν_R and ν^c components respectively, they will break $SU(4)_c \to SU(3)_c$ [31] and mix the field

[11] These normalizations are necessary for the standard model generators T_{SM} of $SU(3)_c$ and $SU(2)_L$ to be normalized in the conventional way: $Tr(T_c^d T_c^e) = \frac{1}{2}\delta^{de}$ and $Tr(T_L^r T_L^s) = \frac{1}{2}\delta^{rs}$ for the fundamental representations, where T_c and T_L are the generators for the $SU(3)_c$ and $SU(2)_L$ groups respectively and $d, e = 1 \ldots 8$.

associated with the remaining $SU(4)_c$ diagonal generator, A_4^{15}, with the field associated with the diagonal generator of $SU(2)_R$, A_R^3, and the $U(1)_\psi$ field A_ψ. The rest of the $SU(4)_c$ and $SU(2)_R$ fields are given square mass proportional to v^2, the sum of the square of the H_R and $\overline{H}_R$ VEVs.

The diagonal generators for the A_4^{15} and A_R^3 fields are T_4^{15} and T_R^3. For the fundamental representations of $SU(4)$ and $SU(2)_R$ respectively [12] :

$$T_4^{15} = \sqrt{\frac{3}{2}}\ diag(\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, -\frac{1}{2}), \qquad T_R^3 = diag(\frac{1}{2}, -\frac{1}{2}).$$

The part of the symmetry breaking G_{4221} to G_{3211} involving the diagonal generators T_4^{15}, T_R^3 and T_ψ is then equivalent to:

$$U(1)_{T_4^{15}} \otimes U(1)_{T_R^3} \otimes U(1)_\psi \to U(1)_Y \otimes U(1)_X.$$

In the rest of this Appendix we explain this particular symmetry breaking in detail. Using the G_{4221} covariant derivative, Eq.21, the covariant derivative for the $U(1)_{T_4^{15}} \otimes U(1)_{T_R^3} \otimes U(1)_\psi$ symmetry is:

$$\begin{aligned} D_\mu &= \partial_\mu + ig_4 T_4^{15} A_{4\mu}^{15} + ig_{2R} T_R^3 A_{R\mu}^3 + \frac{1}{\sqrt{6}} ig_\psi T_\psi A_{\psi\mu} \\ &\equiv \partial_\mu + ig_{B-L} T_{B-L} A_{4\mu}^{15} + ig_{2R} T_R^3 A_{R\mu}^3 + ig_{N\psi} T_\psi A_{\psi\mu} \end{aligned} \tag{24}$$

where $g_{B-L} \equiv \sqrt{\frac{3}{2}} g_4$, $g_{N\psi} \equiv \frac{1}{\sqrt{6}} g_\psi$, $T_{B-L} \equiv \sqrt{\frac{2}{3}} T_4^{15} = \frac{(B-L)}{2}$ and B and L are baryon and lepton number respectively.

In terms of the diagonal generators T_{B-L}, T_R^3 and T_ψ, the ν_R component of H_R and the ν^c component of $\overline{H}_R$ transform under $U(1)_{T_4^{15}} \otimes U(1)_{T_R^3} \otimes U(1)_\psi$ as:

$$\nu_R^H = \left(-\frac{1}{2},\ \frac{1}{2},\ -\frac{1}{2}\right), \qquad \nu_{\overline{H}}^c = \left(\frac{1}{2},\ -\frac{1}{2},\ \frac{1}{2}\right). \tag{25}$$

Therefore, once H_R and $\overline{H}_R$ get their VEVs, the square of the covariant derivative for the A_4^{15}, A_R^3 and A_ψ fields becomes:

$$\left|D_\mu \nu_R^H\right|^2 = \frac{1}{4} v^2 \Big(-\mathbf{g}_{B-L} A_{4\mu}^{15} + \mathbf{g}_{2R} A_{R\mu}^3 - \mathbf{g}_{N\psi} A_{\psi\mu} \Big)^2$$

where $\mathbf{g}_{B-L}$, $\mathbf{g}_{2R}$ and $\mathbf{g}_{N\psi}$ are the g_{B-L}, g_{2R} and $g_{N\psi}$ gauge coupling constants evaluated at the G_{4221} symmetry breaking scale. The above squared covariant derivative can be written in matrix form as:

$$\frac{1}{4} v^2 \begin{pmatrix} A_R^3 & A_4^{15} & A_\psi \end{pmatrix} \begin{pmatrix} \mathbf{g}_{2R}^2 & -\mathbf{g}_{2R}\ \mathbf{g}_{B-L} & -\mathbf{g}_{2R}\ \mathbf{g}_{N\psi} \\ -\mathbf{g}_{2R}\ \mathbf{g}_{B-L} & \mathbf{g}_{B-L}^2 & \mathbf{g}_{B-L}\ \mathbf{g}_{N\psi} \\ -\mathbf{g}_{2R}\ \mathbf{g}_{N\psi} & \mathbf{g}_{B-L}\ \mathbf{g}_{N\psi} & \mathbf{g}_{N\psi}^2 \end{pmatrix} \begin{pmatrix} A_R^3 \\ A_4^{15} \\ A_\psi \end{pmatrix}. \tag{26}$$

Diagonalizing this matrix equation determines the mass eigenstate fields generated by the mixing of the G_{4221} fields A_R^3, A_4^{15} and A_ψ. The 3×3 square mass mixing matrix has two zero eigenvalues and one non-zero eigenvalue so that two massless gauge bosons and one massive gauge boson appear to have been created by the mixing. The massive gauge boson B_H is the following mixture of G_{4221} fields:

$$B_H = \frac{1}{b}\Big(- \mathbf{g}_{2R} A_R^3 + \mathbf{g}_{B-L} A_4^{15} + \mathbf{g}_{N\psi} A_\psi \Big)$$

where $b^2 \equiv \mathbf{g}_{2R}^2 + \mathbf{g}_{B-L}^2 + \mathbf{g}_{N\psi}^2$.

This massive field is an unique mass eigenstate field. However, the degeneracy in the zero-eigenvalue eigenvectors of the square mass mixing matrix implies that all orthogonal combinations of any chosen two massless eigenstate fields also describe two massless eigenstate fields. All the orthogonal combinations of two massless eigenstate fields are physically distinct and so the symmetry breaking mechanism does not generate two unique massless eigenstate fields. We therefore require something in addition to this symmetry breaking mechanism that lifts the degeneracy of the zero-eigenvalue eigenvectors and selects two unique massless gauge fields.

We show below that when we include the low-energy VEV of the S particle from the third generation of the **27** multiplets, the degeneracy in the zero-eigenvalue eigenvectors is lifted and the two massless gauge fields are uniquely chosen to be the gauge field B_Y of the Standard Model hypercharge group and an (effectively massless) gauge field that we call B_X. The B_Y and B_X gauge fields are generated from orthogonal zero-eigenvalued eigenvectors of the above 3×3 square mass mixing matrix and are the following mixture of G_{4221} fields:

$$B_Y = \frac{1}{a}\Big(\mathbf{g}_{B-L} A_R^3 + \mathbf{g}_{2R} A_4^{15} \Big),$$

$$B_X = \frac{1}{ab}\Big(\mathbf{g}_{2R}\mathbf{g}_{N\psi} A_R^3 - \mathbf{g}_{B-L}\mathbf{g}_{N\psi} A_4^{15} + (\mathbf{g}_{2R}^2 + \mathbf{g}_{B-L}^2) A_\psi \Big)$$

where $a^2 \equiv \mathbf{g}_{2R}^2 + \mathbf{g}_{B-L}^2$.

In terms of the diagonal generators T_{B-L}, T_R^3 and T_ψ, the S particle transforms under the $U(1)_{T_4^{15}} \otimes U(1)_{T_R^3} \otimes U(1)_\psi$ symmetry as:

$$S = \Big(0,\ 0,\ 2\Big).$$

The S particle only couples to A_ψ and so its VEV, s, therefore introduces a perturbation proportional to s^2/v^2 to the 33 component of the 3×3 square mass mixing matrix in Eq.26. From Section 3, v is determined to be of the order 10^{16} GeV and we require that $s \sim 10^3$ GeV for EW symmetry breaking.

Diagonalizing the 3×3 square mass mixing matrix with this extremely small perturbation in the 33 component determines the mass eigenstate fields to be the massless

hypercharge gauge field B_Y, and an extremely small mass gauge field and large mass gauge field that can be taken to be the B_X and B_H gauge fields, respectively, in the excellent approximation that $s^2/v^2 = 0$.[12]

It is easy to see why the hypercharge gauge field of the Standard Model is the exact massless gauge field of this symmetry breaking. The hypercharge field is the only massless gauge field generated by the H_R and $\overline{H}_R$ VEVs that does not contain the A_ψ field and therefore the only massless gauge field that S does not couple to. If the A_ψ field is removed from the G_{4221} symmetry then the mixing of the remaining G_{4221} diagonal generators becomes equivalent to $U(1)_{T_4^{15}} \otimes U(1)_{T_R^3} \to U(1)_Y$ when H_R and $\overline{H}_R$ get VEVs [9,31].

The mass eigenstate fields B_Y, B_X and B_H can be written in terms of the G_{4221} fields A_R^3, A_4^{15} and A_ψ in the following matrix form:

$$\begin{pmatrix} B_Y \\ B_X \\ B_H \end{pmatrix} = \begin{pmatrix} \mathbf{g}_{B-L}/a & \mathbf{g}_{2R}/a & 0 \\ \mathbf{g}_{2R}\mathbf{g}_{N\psi}/ab & -\mathbf{g}_{B-L}\mathbf{g}_{N\psi}/ab & (\mathbf{g}_{2R}^2 + \mathbf{g}_{B-L}^2)/ab \\ -\mathbf{g}_{2R}/b & \mathbf{g}_{B-L}/b & \mathbf{g}_{N\psi}/b \end{pmatrix} \begin{pmatrix} A_R^3 \\ A_4^{15} \\ A_\psi \end{pmatrix}. \quad (27)$$

We choose to parameterize this orthogonal 3×3 matrix in terms of rotation and reflection matrices in the following way:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ c_{23}s_{12} & -c_{23}c_{12} & s_{23} \\ -s_{23}s_{12} & s_{23}c_{12} & c_{23} \end{pmatrix}$$

where $c_{12} = \mathbf{g}_{B-L}/a$, $s_{12} = \mathbf{g}_{2R}/a$, $c_{23} = \mathbf{g}_{N\psi}/b$ and $s_{23} = a/b$. The mixing angles θ_{12} and θ_{23} are therefore given by $\tan\theta_{12} = \mathbf{g}_{2R}/\mathbf{g}_{B-L}$ and $\tan\theta_{23} = a/\mathbf{g}_{N\psi}$.

Taking the transpose of Eq.27, the G_{4221} fields A_R^3, A_4^{15} and A_ψ can be written in terms of the mass eigenstate fields B_Y, B_X and B_H as:

$$\begin{pmatrix} A_R^3 \\ A_4^{15} \\ A_\psi \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12}c_{23} & -s_{12}s_{23} \\ s_{12} & -c_{12}c_{23} & c_{12}s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} B_Y \\ B_X \\ B_H \end{pmatrix}.$$

Putting this matrix equation into the covariant derivative for the $U(1)_{T_4^{15}} \otimes U(1)_{T_R^3} \otimes U(1)_\psi$ symmetry, Eq.24, determines the covariant derivative for the massless gauge fields B_Y and B_X to be:

$$D_\mu = \partial_\mu + ig_Y Y B_{Y\mu} + ig_X^0 X B_{X\mu}$$

where:

$$Y = T_R^3 + T_{B-L} = T_R^3 + (B-L)/2$$

[12] We have ignored the VEV of the Standard Model Higgs boson in this symmetry breaking.

is the Standard Model hypercharge [31],

$$X = (T_\psi + T_R^3) - c_{12}^2 Y \tag{28}$$

is the non-normalized charge of the B_X gauge field. g_Y and g_X^0 are the non-normalized universal gauge coupling constants of the B_Y and B_X fields respectively and, at the G_{4221} symmetry breaking scale, are given by Eq.29 and Eq.30:[13]

$$g_Y = \frac{\mathbf{g}_{2R}\ \mathbf{g}_{B-L}}{a} \tag{29}$$

$$g_X^0 = \frac{a}{b}\mathbf{g}_{N\psi}. \tag{30}$$

Eq.29 and Eq.30 can be written is terms of $\alpha_Y = \frac{g_Y^2}{4\pi}$ and $\alpha_X^0 = \frac{(g_X^0)^2}{4\pi}$, see Eq.15 and Eq.13 in Section 3.

The charges X and Y are not E_6 normalized. We write the normalized respective charges as T_X and T_Y where:

$$T_X = X/N_X, \qquad T_Y = Y/N_Y$$

and the normalization constants N_X and N_Y are given by:

$$N_X^2 = 7 - 2c_{12}^2 + \frac{5}{3}c_{12}^4, \qquad N_Y^2 = \frac{3}{5}$$

Note that the Abelian generator T_Y is just the conventional GUT normalized hypercharge. T_X and T_Y have been E_6 normalized using Eq.22 which is equivalent to:

$$\sum_{\mathbf{27}} T_Y^2 = \sum_{\mathbf{27}} T_X^2 = 3$$

where the sum is over all the $G_{3211} \equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ representations of the fundamental **27** E_6 multiplet and $U(1)_X$ is the unitary group of the B_X field.[14]

In terms of the E_6 normalized charges T_X and T_Y, the covariant derivative for the B_X and B_Y gauge fields becomes:

$$D_\mu = \partial_\mu + ig_1 T_Y B_{Y\mu} + ig_X T_X B_{X\mu} \tag{31}$$

[13] Note that Eq.29 is the relation that g_Y must satisfy if the Pati-Salam symmetry, without the $U(1)_\psi$, was broken to the standard model gauge group using a Higgs boson that transforms as $(4, 1, 2)$ and gets a VEV in the ν_R direction [9].

[14] We could have defined X and N_X differently as long as T_X is the same. Here we have chosen to define X so that it can be written in terms of hypercharge Y.

where g_1 and g_X are the *normalized* universal gauge coupling constants of the B_Y and B_X fields respectively. At the G_{4221} symmetry breaking scale, the normalized gauge coupling constants g_1 and g_X are the following combinations of G_{4221} gauge coupling constants:

$$g_1 = N_Y \frac{\mathbf{g}_{2R}\ \mathbf{g}_{B-L}}{a}, \qquad g_X = N_X\ \frac{a}{b}\mathbf{g}_{N\psi}.$$

From Eq.28, the charge T_X of the $U(1)_X$ group depends on the Pati-Salam gauge coupling constants g_{2R} and g_{B-L} evaluated at the G_{4221} symmetry breaking scale. Therefore, under the excellent approximation that $s^2/v^2 = 0$, a massless gauge boson exists that couples to particles with a charge that depends on the values that certain coupling constants take at some high energy scale. Although this may be unusual, it does not appear to pose any problems. Indeed, like any other quantum charge, T_X is a dimensionless constant that is independent of the energy scale at which the interaction between the particle and the A_X field occurs and, although the numbers that X takes may not be able to be arranged into fractions like Y, they are still discrete and sum to zero for a complete E_6 representation. However, unlike conventional $U(1)$ charges, T_X is obviously very model dependent since different E_6 models with an intermediate Pati-Salam symmetry will, in general, contain different values of the gauge coupling constants g_{2R} and g_4 evaluated at the G_{4221} symmetry breaking scale. It is easy to prove that it is a general rule that, if three massless gauge fields are mixed, then at least two of the resulting mass eigenstate fields must have a charge that depends on the value of the original gauge coupling constants. Therefore this gauge coupling dependence is not peculiar to the Higgs symmetry breaking mechanism discussed in this Appendix, but to any symmetry breaking mechanism involving three fields.

In this Appendix we have illustrated how the $G_{4221} \equiv SU(4) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_\psi$ symmetry can be broken to the symmetry $G_{3211} \equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$ when the G_{4221} multiplets H_R, $\overline{H}_R$ and S get vacuum expectation values. Using the covariant derivatives for the G_{4221} symmetry, Eq.21, and the $U(1)_Y \otimes U(1)_X$ symmetry, Eq.31, the covariant derivative for the G_{3211} symmetry is given by:

$$D_\mu = \partial_\mu + ig_3 T^n_{3c} A^n_{3c\mu} + ig_{2L} T^s_L A^s_{L\mu} + ig_1 T_Y B_{Y\mu} + ig_X T_X B_{X\mu} \tag{32}$$

where A^n_{3c} and T^n_{3c} are the $SU(3)_c$ fields and generators derived from the $SU(4)$ symmetry respectively (with $n = 1 \ldots 8$) and g_{3c} is the universal gauge coupling constant of A^n_{3c}.

We consider this G_{3211} symmetry as an effective high energy symmetry under the assumption that the low-energy VEVs of the MSSM singlet S and MSSM Higgs bosons can be neglected at higher energy scales.

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